1. Solve the following first order differential equation using the power series method (about $x_0 = 0$), give all recurrence relations, and verify that a_0 is the arbitrary initial condition. Also, use techniques from earlier in the semester to solve the equation and show that the two solutions are equivalent.

$$(1 - x)\dot{y} = y.$$

2. The Tchebycheff (1821-1894) differential equation is

$$(1 - x^2)\ddot{y} - x\dot{y} + \alpha^2 y = 0,$$

where α is a constant.

a. Find the recurrence relations, then determine two linearly solutions in powers of x for $|x| < 1$.

b. Show that if α is a nonnegative integer n, then there is a polynomial solution of degree n. These polynomials, when properly normalized, are called the Tchebycheff polynomials and are useful in problems requiring polynomial approximation to a function defined on $-1 \leq x \leq 1$.

c. Find the Tchebycheff polynomials (not necessarily normalized) for $\alpha = 0$, 1, 2, and 3.