1. Solve the following first order differential equation using the power series method (about \( x_0 = 0 \)),
give all recurrence relations, and verify that \( a_0 \) is the arbitrary initial condition. Also, use techniques
from earlier in the semester to solve the equation and show that the two solutions are equivalent.

\[
(1 - x)\dot{y} = y.
\]

2. The Tchebycheff (1821-1894) differential equation is

\[
(1 - x^2)\ddot{y} - x\dot{y} + \alpha^2 y = 0,
\]

where \( \alpha \) is a constant.

a. Find the recurrence relations, then determine two linearly solutions in powers of \( x \) for \(|x| < 1\).

b. Show that if \( \alpha \) is a nonnegative integer \( n \), then there is a polynomial solution of degree \( n \). These
polynomials, when properly normalized, are called the Tchebycheff polynomials and are useful in
problems requiring polynomial approximation to a function defined on \(-1 \leq x \leq 1\).

c. Find the Tchebycheff polynomials (not necessarily normalized) for \( \alpha = 0, 1, 2, \) and \( 3 \).