

1. Consider the following initial value problems:

a. $\frac{dy}{dt} = 0.3y, \quad y(0) = 20.$

b. $\frac{dy}{dt} = 10 - 0.3y, \quad y(0) = 10.$

Solve each of these initial value problems, then use Euler's method to approximate the solution using a stepsize of $h = 0.2$ for $t \in [0, 1]$. Find the approximate solution for $y(1)$, then compute the percent error between the actual solution and the approximate solution using Euler's method.

2. A population of animals that includes emigration satisfies the differential equation

$$P' = kP - m, \quad P(0) = 100,$$

where $k = 0.1$ and $m = 2$.

a. Solve this differential equation and find $P(1)$.

b. Use Euler's method with $h = 0.2$ to approximate the solution at $t = 1$. Find the percent error between the actual solution and this approximate solution at $t = 1$.

3. The temperature of an object is initially 50°C . If it is in a room where the temperature, $T_e(t)$, is slowly decreasing with $T_e(t) = 20 - t$, then using Newton's Law of Cooling, the temperature of the object satisfies the differential equation

$$T' = -k(T - (20 - t)),$$

where $k = 0.2 \text{ hr}^{-1}$.

a. Verify that the solution to this initial value problem is given by

$$T(t) = 25 - t + 25e^{-0.2t}$$

and find the temperature at $t = 2$.

b. Use Euler's method with $h = 0.5$ to approximate the solution at $t = 2$. This means that you are to solve the differential equation above and take four Euler's method steps. Find the percent error between the actual solution and this approximate solution at $t = 2$.

4. The body temperature of a particular animal is normally 40°C . Suppose this animal is hit by a car at midnight ($t = 0$), and the environmental temperature, $T_e(t)$, over the next few hours is slowly decreasing with $T_e(t) = 15 - t$. From Newton's Law of Cooling, the temperature of the roadkill satisfies the differential equation

$$T' = -k(T - (15 - t)),$$

where $k = 0.2 \text{ hr}^{-1}$.

a. Verify that the solution to this initial value problem is given by

$$T(t) = 20 - t + 20e^{-0.2t},$$

and find the temperature of the body at 2 AM.

b. Use Euler's Method with $h = 0.5$ to approximate the temperature at $t = 2$. This means that you are to use the differential equation above and take four Euler's method steps.