1. a. A ball thrown vertically with an initial velocity of 64 ft/sec upwards satisfies the equation

$$h(t) = 64t - 16t^2,$$

with $h(t)$ being the height of the ball. Plot a graph of the height of the ball as a function of time, t. Find when the ball hits the ground. Also, determine the maximum height of the ball and when this occurs.

b. Find the average velocity in the intervals $t \in [0, 1]$, $[1, 2]$, and $[2, 4]$.

c. Approximate the velocity at $t = 1$ by finding the average velocity in the intervals $t \in [1, 1.1]$, $[1, 1.01]$, and $[1, 1.001]$.

2. A ball is tossed into the air with an initial velocity of 48 ft/sec from a 64 ft platform. Its height, h (in ft), above the ground t seconds after it is thrown is given by

$$h(t) = -16t^2 + 48t + 64.$$

a. Find the average velocity of the ball for the first two seconds. Also, find the average velocity between times $t = 2$ and $t = 2.5$.

b. Sketch a graph of the flight of the ball. What is the maximum height of the ball and when does it occur?

c. When does the ball hit the ground and what is its velocity then? (Hint: Find the slope of the tangent line at the time when the ball hits the ground.)

3. Suppose that an object shot vertically from a 58.8 m tall building satisfies the height function:

$$h(t) = 58.8 + 19.6t - 4.9t^2,$$

where t is in seconds and h is in meters.

a. Find the average velocity between $t = 0$ and $t = 1$ sec. Repeat this calculation for $t \in [1, 2]$, $t \in [2, 3]$, $t \in [3, 4]$, and $t \in [4, 5]$.

b. Determine the maximum height of the object and when the object hits the ground ($h(t) = 0$). Sketch a graph of the height of the object.

c. Find the velocity of the object at $t = 4$ sec by computing the average velocity between $t = 4$ and $t = 4 + \tau$, then letting $\tau \to 0$.

4. a. A cat is sitting on a ledge 12 ft above the ground. A bird flies by at a height of 18 ft above the ground. The cat leaps up with a vertical velocity of 16 ft/sec trying to catch the bird. If we ignore air resistance and use an acceleration from gravity of -32 ft/sec^2, then the height of the cat above the ground, $h(t)$, is given by the formula

$$h(t) = 12 + 16t - 16t^2.$$

Find the maximum height that the cat achieves and how long it takes to reach that maximum height. Can the cat catch the bird?

b. Find the average velocity of the cat for the intervals $t \in [0, \frac{1}{2}]$, $t \in [\frac{1}{2}, 1]$, and $t \in [1, \frac{3}{2}]$.

c. Determine the time when the cat hits the ground and the velocity of impact. Sketch a graph of the height of the cat as a function of t.

5. A kangaroo can leap vertically 240 cm. The initial velocity, v_0 is unknown, so we want to determine it from the data on how high it can jump using Newton's law of gravity. The equation describing the height of the kangaroo is

$$h(t) = v_0 t - 490t^2.$$

a. Use the information above to determine the animal's initial upward velocity, v_0, then find how long the kangeroo is in the air.

b. Find the average velocity of the kangaroo between $t = 0$ and $t = 1$.

6. Consider the function $f(x) = 1 - x^2$. To find the equation of the tangent line at the point $x = 1$ or $(1, 0)$, we find a sequence of secant lines passing through $(1, 0)$.

a. Let one point on all secant lines be $(1, 0)$. The other points in the sequence have $x = 2$, $x = 1.5$, $x = 1.1$, and $x = 1.01$. Find the sequence of secant lines with these points on the line and on $f(x)$. Sketch a graph of $f(x)$ and the secant lines.

b. Use these secant lines to predict the equation of the tangent line. The slope of the tangent line gives the derivative at $x = 1$, so find the derivative of $f(x)$ at $x = 1$.

7. Consider the function $f(x) = 2x - x^2$. To find the equation of the tangent line at the point $x = 0$ or $(0, 0)$, we find a sequence of secant lines passing through $(0, 0)$.

a. Let one point on all secant lines be $(0, 0)$. The other points in the sequence have $x = 1$, $x = 0.5$, $x = 0.1$, and $x = 0.01$. Find the sequence of secant lines with these points on the line and on $f(x)$. Sketch a graph of $f(x)$ and the secant lines.

b. Use these secant lines to predict the equation of the tangent line. The slope of the tangent line gives the derivative at $x = 0$, so find the derivative of $f(x)$ at $x = 0$.

8. Consider the functions below. Find the equation of the secant line through the points $(1, f(1))$ and $(1 + h, f(1 + h))$ for each of these functions. Let h get small and determine the slope of the tangent line through $(1, f(1))$, which gives the value of the derivative of $f(x)$ at $x = 1$.

a. $f(x) = x^2 + 2$,

b. $f(x) = 3x - x^2$,

c. $f(x) = \sqrt{2x + 2}$,

d. $f(x) = 3x - 4$,

e. $f(x) = \dfrac{1}{x + 1}$,

f. $f(x) = \dfrac{3}{2 - x}$.