Find the derivative for each of the following functions.

1. $f(x) = x^4 + 7x^3 - 2x^2 - 4x + 3,$

2. $g(x) = 3x^2 - 3x + 4 - 2x^{-3},$

3. $h(t) = t^3 - 5t + \frac{1}{2} - \frac{1}{t^2},$

4. $k(z) = \frac{3z^2}{2} + 6z - \sqrt{z},$

5. $p(z) = z^{\frac{1}{3}} + 4.7z^2 - 7\sqrt{z^5},$

6. $q(w) = 3w^{-0.4} + 2.1w^5 - \frac{2}{\sqrt{w}},$

7. $f(x) = ax^2 + bx + c,$

8. $g(x) = A - \frac{B}{x^3} + \frac{C}{\sqrt{x}} - Dx^4.$

9. In the linear section, we found that the growth of a child satisfies the equation

$$h(a) = 6.46a + 72.3,$$

where the age, a, is in years and the height, h, is in cm.

a. Find dh/da. What is the growth rate at age 2? At age 6?

b. If a child is 135 cm at age 10, what is the predicted height at age 11?

10. The lecture notes showed that the number of species of herpatofauna, N on Caribbean Islands as a function of the area in square miles, A, is approximated by the formula

$$N = 3A^{\frac{1}{3}}.$$

a. Find the rate of change in number of species as a function of area, dN/dA, when the area of the island is 64, 125, and 1000 square miles.

b. Sketch a graph of the derivative, dN/dA, for $0 \leq A \leq 1000$.

11. A ball falling under the influence of gravity without air resistance satisfies the equation

$$y(t) = -4.9t^2,$$

where y is in meters and t is in seconds.

a. Find an expression for the velocity, $v(t) = y'(t)$.

b. What is the velocity at $t = 1$ and $t = 5$?

12. A ball that is thrown vertically falling under the influence of gravity without air resistance from a 128 ft platform with an upward velocity of 32 ft/sec satisfies the equation

$$h(t) = 128 + 32t - 16t^2,$$

where h is in feet and t is in seconds.

a. Find an expression for the velocity, $v(t) = h'(t)$. Determine when the velocity is zero, then determine the maximum height of the ball. What is the velocity at $t = 2$ and $t = 4$.

b. Sketch a graph of $h(t)$, showing crucial points, including the h-intercept, the maximum height, and when the ball hits the ground.

13. Suppose that a population of insects, P (individuals), in a controlled experiment with constant food supply is shown to have a growth rate that fits the logistic growth function

$$g(P) = 0.04P(1 - P/800),$$

where the time is measured in days.

a. Find the population when the growth rate $g(P)$ is zero (the P-intercepts). This gives the equilibria for this experiment. Give a biological interpretation for each of the equilibria.

b. Compute the derivative of $g(P)$, then determine when $g(P)$ has a maximum growth rate. (State the units for the growth rate.) Give a biological interpretation of this maximum growth rate. Sketch the graph of $g(P)$.

14. An experiment with a sample of paramecium, P (individuals/ml), with a fixed food supply is shown to satisfy the logistic growth function

$$G(P) = 0.02P(1 - 0.025P),$$

where the time is in hours.

a. Find the equilibria for this population, *i.e.*, when the growth rate $G(P)$ is zero. Give a biological interpretation for each of the equilibria.

b. Compute the derivative of $G(P)$, then determine when $G(P)$ has a maximum growth rate. (State the units for this growth rate.) Give a biological interpretation of this maximum growth rate. Sketch the graph of $G(P)$.

15. A cat is crouching on a ledge that is 12 feet above the ground, trying to ambush pigeons that fly by.

a. Suppose that a pigeon flies by 4 feet above the cat, and that the cat jumps off the ledge with just enough vertical velocity, v_0 to catch the pigeon. If the height of the cat is given by

$$h(t) = -16t^2 + v_0 t + 12,$$

then find the velocity $v(t) = h'(t)$ of the cat at any time, $t \geq 0$.

b. Find when the velocity is equal to zero in terms of v_0. This is the time at the maximum height.

c. Since the cat is 16 ft in the air at this time, use the equation for the height of the cat, $h(t)$ to compute the initial velocity of the cat, v_0. Substitute this into the velocity equation, $v(t)$ to give the velocity of the cat at any time between jumping and hitting the ground. What is the velocity of the cat after 1 second?

d. Find when the cat hits the ground with the pigeon and what is the velocity of the cat that it hits the ground.

16. a. Lizards are cold-blooded animals whose temperatures roughly match the surrounding environment. Suppose the body temperature, $T(t)$, of a lizard is measured for a period of 18 hours from midnight until 6 PM. The body temperature (in °C) of the lizard over this period of time (in hours) is found to be well approximated by the polynomial

$$T(t) = -0.01t^3 + 0.285t^2 - 1.80t + 15.$$

Find the general expression for the rate of change of body temperature per hour ($\frac{dT}{dt}$).

b. Use this information to find what the rate of change of body temperature is at midnight, 4 AM, 8 AM, noon, and 4 PM. Which of these times gives the fastest increase in the body temperature and which shows the most rapid cooling of the lizard?