1. Consider the functions $f(t) = 2t^2 + t$ and $g(t) = t - 2$.

a. Evaluate $f(0)$, $f(2)$, $g(-2)$, and $g(3)$.

b. Create the composite functions $f(g(t))$ and $g(f(t))$ and write the expressions in the simplest forms.

c. Evaluate $f(g(1))$ and $g(f(1))$.

2. Consider the function $f(x) = 30 + x - x^2$.

a. What is the range of this function (assuming a domain of all x)?

b. Find the domain of $f(x)$, if the range of f is restricted to $f(x) > 0$.

Solve the following quadratic equations (if possible):

3. $x^2 + 8x + 15 = 0$

4. $x^2 + 4x - 3 = 0$

5. $2x^2 - 3x + 1 = 0$

6. $x^2 - x - 20 = 0$

7. $x^2 + 3x - 5 = 0$

8. $x^2 - 9 = 0$

9. $2x^2 - 5x = 0$

10. $x^2 - 2x + 2 = 0$

In each of the following problems, sketch the pair of functions on a single graph. Find the x and y-intercepts for both functions. What is the slope of the line? Find the coordinates of the vertex of the parabola. Finally, determine the coordinates of the points of intersection of these curves.

11. $f(t) = 4 - t^2$ and $g(t) = t - 2$.

12. $f(x) = x^2 + 4x + 4$ and $g(x) = 4 - 2x$.

Word Problems:

13. A ball is thrown vertically with a velocity of 48 ft/sec from a platform that is 64 ft in the air. The height of the ball satisfies the equation:

$$h(t) = 64 + 48t - 16t^2.$$

a. Sketch a graph of $h(t)$ vs. t.

b. Find the maximum height of the ball, then determine when the ball hits the ground.

14. Acetic acid arises in the bacterial breakdown of many fruits often resulting in vinegar. The equilibrium constant (ionization constant) for acetic acid is $K_a = 1.75 \times 10^{-5}$. Use the information developed in the notes for formic acid as a guide to determine the concentration of $[H^+]$ and pH of 0.1N and 1N solutions of acetic acid.

15. A rectangle with a length x and width y has a perimeter of 40 cm.

a. Write an expression for the width y as a function of the length x, using this information.

b. The area of a rectangle is $A = xy$. Substitute the expression for y into this formula for the area to produce a function of the area as a function of x alone.

c. Sketch a graph of the area as a function of x and determine what value of x produces the largest area. What geometric figure does this produce?

16. The braking distance d (in feet) of a car is divided into two components. One part depends on reaction time. The number of feet for reaction time is about the same as as the speed of the car in miles/hour, v. The other component is due to friction, which is a force that is proportional to the velocity squared. Adding these two components together, we find the braking distance satisfies the equation

$$d = v + \frac{v^2}{20}.$$

Find the braking distance at 60 miles/hr. Also, determine all velocities that result in a braking distance that is less than 75 ft.

17. For animals that reproduce seasonally, we find that their population satisfies a difference equation

$$P_{n+1} = P_n + g(P_n),$$

where P_n is the population in the nth season and $g(P)$ (in individuals per generation) is the growth rate of the population. This equation simply says that the population in the next generation is equal to the population of the previous generation plus the net growth of the population over the last season.

a. Suppose that the growth rate $g(P)$ satisfies the quadratic equation

$$g(P) = 0.02P - 0.000025P^2.$$

Sketch a graph of this growth rate function.

b. The population is at equilibrium when the growth rate is zero. Find the equilibrium populations.

c. The growth rate is at a maximum at the vertex of parabola. Find the population that produces this maximum growth rate and what that growth rate is.

18. The Lambert-Beer law for absorbance of light by a spectrophotometer is a linear relationship, which can have the form

$$A = mc,$$

where c is the concentration of the sample, A is absorbance, and m is the slope that must be determined from experiments.

a. Below are data collected on samples from a collection of acid standards using an acid indicator.

c (mM)	1	2	5
A	2.0	4.1	9.8

Write a formula for the quadratic function $J(m)$ that measures the sum of squares error of the line fitting the data. Find the vertex of this quadratic function. This gives the value of the best slope m, while the $J(m)$ value of the vertex gives the least sum of squares error.

b. Use this model (with the best value of m) to determine the concentration of an unknown acid with absorbances of $A = 3.5$ and 6.2.

19. In looking through some old photos, a woman finds a picture of her great-grandfather standing near the family home, where she now lives. In the photograph, she measures the height of the roofline, which she knows to be 20 ft, as 3.3 cm. The 2 ft wide window measures 0.5 cm on the photo, and the distance from the front door to the oak tree at the driveway is 12 feet, which is 2 cm in the photograph.

a. The conversion of measurements in the photo p to measurements in actual distance d is given by the formula

$$d = kp.$$

Write a formula for the quadratic function $J(k)$ that measures the sum of squares error of the line fitting the measurements in the photo. Find the vertex of this quadratic function. This gives the value of the best slope k, while the $J(k)$ value of the vertex gives the least sum of squares error.

b. In the photograph, her great-grandfather is 1 cm tall. Her mother remembers her grandfather as a tall man of about 6 ft , whereas her father thinks he was shorter, about 5 ft 6 inches (5.5 ft). Use the model (with the best value of k) to predict the height of the great-grandfather and determine whether the mother or father better remembers the height of her great-grandfather.