Find the derivatives of the following functions:

1. $f(x) = (x^3 - 3x^2 + 7)(x^4 - 2x^2 + 6x - 1)$,

2. $f(x) = (x^2 - e^{2x} + 1)(3x + 8)$,

3. $f(x) = x^2 e^{-x} + 21\sqrt{x}$,

4. $f(x) = \frac{1}{x^2} \ln(x) - e^{2x}(x^2 - 1)$.

Find the derivative and sketch the curves of the functions below. Give the domain of each of the functions. List all maxima and minima for each graph. Also, give the $x$ and $y$-intercepts and any asymptotes if they exist.

5. $y = 3xe^{-0.02x}$,

6. $y = (x - 2)e^{-x}$,

7. $y = \frac{1}{x} \ln(x)$,

8. $y = (x^2 - 3)e^{x}$,

9. $y = x^2 \ln(x)$.

10. A tumor growing according to Gompertz's model satisfies the growth law

$$G(N) = N(0.8 - 0.04\ln(N)) \text{ (cells/day)},$$

where $N$ is the number of tumor cells and the time units are days.

a. Find the equilibrium number of tumor cells by solving when $G(N) = 0$.

b. Compute $G'(N)$ and determine the population at which the maximum rate of growth of the tumor is occurring. What is the maximum growth rate (in cells/day)?

11. Many biologists in fishery management use Ricker's model to study the population of fish. A general expression for the growth of the population of fish is given by

$$R(P) = aPe^{-bP} - (h+1)P,$$

where $a$ and $b$ are parameters that fit the population dynamics of the fish and $h$ is the harvesting level of the fisheries. Suppose that the best fit to a set of data for the number of fish sampled from a particular river gives

$$R(P) = 5Pe^{-0.002P} - 1.5P,$$

where $P$ is in fish/100 m of river and $R(P)$ has units of fish/100 m/day.

a. Find the equilibrium population of the fish by finding when $R(P) = 0$.

b. Find the rate of change in the growth rate by computing $R'(P)$.

c. Evaluate $R'(P)$ at $P = 200$, 250, and 300. What are the units for these calculations. Give a biological interpretation of these results.

12. The distribution of seeds from around a plant satisfies an exponential distribution. It is more likely to find seeds close to a plant than it is far away. Suppose that experimental measurements fitting the seed distribution radially from the plant satisfies

$$P(r) = 0.04r\, e^{-0.2r},$$

where $P$ is the probability of finding a seed $r$ meters from the plant. Find the distance $r$ and the probability $P(r)$ at which a seed is most likely to land. That is find the maximum probability from the function above. Sketch a graph of $P(r)$ showing any intercepts, asymptotes, and local extrema.

13. When coughing, the windpipes contract to increase the velocity of air passing through the windpipe to help clear mucus. The velocity, $v$, at which the air flows through the windpipe depends on the radius, $r$ of the windpipe. If $R$ is the resting radius of the windpipe, then the velocity of air passing through the windpipe satisfies:

$$v(r) = Ar^2(R - r),$$

where $A$ is a constant dependent on the strength of the diaphram muscles. Find the value of $r$ that maximizes the velocity of air and determine the velocity of the air flowing through the windpipe.