1. Let $y_{n+1} = 0.7y_n + 6$ with $y_0 = 10$. Then

$$\begin{aligned} y_1 &= 0.7y_0 + 6 = 0.7(10) + 6 = 13 \\ y_2 &= 0.7y_1 + 6 = 0.7(13) + 6 = 15.1 \\ y_3 &= 0.7y_2 + 6 = 0.7(15.1) + 6 = 16.57 \end{aligned}$$

Equilibrium occurs when $y_{n+1} = y_n = y_e$, so $y_e = 0.7y_e + 6$ or $0.3y_e = 6$. Thus, $y_e = 20$. The solution is approaching the equilibrium, so it is stable.

3. This breathing model is given by

$$c_{n+1} = (1-q)c_n + q\gamma = 0.82c_n + 0.18(0.78),$$

$c_0 = 0.7$. Then

$$\begin{aligned} c_1 &= 0.82c_0 + 0.1404 = 0.82(0.7) + 0.1404 = 0.7144 \\ c_2 &= 0.82c_1 + 0.1404 = 0.82(0.7144) + 0.1404 = 0.7262 \\ c_3 &= 0.82c_2 + 0.1404 = 0.82(0.7262) + 0.1404 = 0.7359 \end{aligned}$$

Equilibrium occurs when $c_{n+1} = c_n = c_e$, so $c_e = 0.82c_e + 0.1404$ or $0.18c_e = 0.1404$. Thus, $c_e = 0.78$. The solution is approaching the equilibrium, so it is stable.

5. From the mathematical model $c_{n+1} = (1-q)c_n + q\gamma$ with that $c_0 = 0.68$ and $c_1 = 0.694$, it follows that

$$0.694 = (1-q)0.68 + 0.78q.$$

Therefore, it follows that

$$(0.78 - 0.68)q = 0.694 - 0.68 \qquad \text{or} \qquad q = 0.14.$$

The functional reserve capacity, given that $V_i = 400$ ml, satisfies

$$V_r = \frac{(1-q)V_i}{q} = \frac{0.86(400)}{0.14} = 2457 \text{ ml}.$$

6. a. From the mathematical model $c_{n+1} = (1-q)c_n + q\gamma$ with that $c_0 = 30$ and $c_1 = 25.8$, it follows that

$$25.8 = (1-q)30 + 5.2q.$$

Therefore, it follows that

$$(30 - 5.2)q = 30 - 25.8 \qquad \text{or} \qquad q = 0.16935.$$

The functional reserve capacity, V_r, given that $V_i = 350$ ml, satisfies

$$V_r = \frac{(1-q)V_i}{q} = \frac{0.83065(350)}{0.16935} = 1717 \text{ ml}.$$

b. To find the expected concentration of Helium in this patient's 10^{th} breath, c_{10}, the model is iterated as follows:

$$\begin{aligned}
c_1 &= 0.83065c_0 + 0.16935(5.2) = 0.83065(30) + 0.88062 = 25.8 \\
c_2 &= 0.83065c_1 + 0.88062 = 0.83065(25.8) + 0.88062 = 22.31 \\
c_3 &= 0.83065c_2 + 0.88062 = 0.83065(22.31) + 0.88062 = 19.41 \\
c_4 &= 0.83065(19.41) + 0.88062 = 17.01 \\
c_5 &= 0.83065(17.01) + 0.88062 = 15.01 \\
c_6 &= 0.83065(15.01) + 0.88062 = 13.35 \\
c_7 &= 0.83065(13.35) + 0.88062 = 11.97 \\
c_8 &= 0.83065(11.97) + 0.88062 = 10.82 \\
c_9 &= 0.83065(10.82) + 0.88062 = 9.87 \\
c_{10} &= 0.83065(9.87) + 0.88062 = 9.08
\end{aligned}$$

Equilibrium occurs when $c_{n+1} = c_n = c_e$, so $c_e = 0.83065c_e + 0.88062$ or $0.16935c_e = 0.88065$. Thus, $c_e = 5.2$ ppm. The solution is approaching the equilibrium, so it is stable.

c. Since the functional reserve capacity is equal to the expiratory reserve, V_{exp}, volume plus the residual volume, V_{res},

$$V_{exp} = V_r - V_{res} = 2457 - 950 = 767 \text{ ml.}$$

Since the vital capacity, V_C, is equal to the sum of the tidal volume and the inspiratory and expiratory reserve volumes,

$$V_{insp} = V_C - V_i - V_{exp} = 1300 - 350 - 767 = 183 \text{ ml.}$$

The normal expiratory reserve for a woman is about 800-1000 ml, so this value is a little low. The normal inspiratory reserve volume for a woman is about 2500 ml, so this patient has an extremely low value for this parameter, suggesting problems in the diaphragm muscles (polio or spinal paralysis).

8. The population in the U. S. (in millions) is given by

Year	1900	1910	1920	1930
Population	76.0	92.0	105.7	122.8

a. The general solution to the discrete Malthusian growth model is

$$p_{n+1} = 76.0(1.17)^n.$$

It follows that $p_1 = 76.0(1.17) = 88.9$ (million), $p_2 = 76.0(1.17)^2 = 104.0$ (million), and $p_3 = 76.0(1.17)^3 = 121.7$ (million). The errors are given by

$$\begin{aligned}
100\frac{(88.9 - 92.0)}{92.0} &= -3.3\%, \\
100\frac{(104.0 - 105.7)}{105.7} &= -1.6\%, \\
100\frac{(121.7 - 122.8)}{122.8} &= -0.88\%.
\end{aligned}$$

b. For the Malthusian growth model with immigration of 3.0 million, then the model is given by

$$p_{n+1} = 1.14p_n + 3.0,$$

where n is in decades. Starting with $p_0 = 76.0$ million, we have

$$\begin{aligned} p_1 &= 1.14p_0 + 3.0 = 1.14(76.0) + 3.0 = 89.6 \text{ (million)} \\ p_2 &= 1.14p_1 + 3.0 = 1.14(89.6) + 3.0 = 105.2 \text{ (million)} \\ p_3 &= 1.14p_2 + 3.0 = 1.14(105.2) + 3.0 = 122.9 \text{ (million)} \end{aligned}$$

The errors are given by

$$\begin{aligned} 100\frac{(89.6 - 92.0)}{92.0} &= -2.6\%, \\ 100\frac{(105.2 - 105.7)}{105.7} &= -0.48\%, \\ 100\frac{(122.9 - 122.8)}{122.8} &= 0.09\%. \end{aligned}$$

10. a. The population of a species of moth satisfies the model

$$P_{n+1} = rP_n + \mu.$$

From the data in 1990, 1991, and 1992 with populations of $P_0 = 6000$, $P_1 = 5500$, and $P_2 = 5100$, respectively, the model gives the following:

$$\begin{aligned} P_{n+1} &= rP_n + \mu \\ 5500 &= 6000r + \mu \\ 5100 &= 5500r + \mu. \end{aligned}$$

By substracting these equations, we have $5500 - 5100 = (6000 - 5500)r$, so

$$r = \frac{400}{500} = 0.8.$$

But $5500 = 6000(0.8) + \mu$, so

$$\mu = 700.$$

Therefore, we can find the number of moths in the following 3 years (1993, 1994, and 1995) with the following computation:

$$\begin{aligned} p_3 &= 0.8p_2 + 700 = 0.8(5100) + 700 = 4780 \\ p_4 &= 0.8p_3 + 700 = 0.8(4780) + 700 = 4524 \\ p_5 &= 0.8p_4 + 700 = 0.8(4524) + 700 = 4319. \end{aligned}$$

b. As before, the equilibrium satisfies $P_{n+1} = P_n = P_e$, so $P_e = 0.8P_e + 700$ or $0.2P_e = 700$, which gives

$$P_e = 3500.$$

This population is moving towards the equilibrium, so it is considered stable. The equilibrium population is the long time behavior of the model in this case, so ultimately, we expect 3500 moths on the island.

c. The graph of the updating function and the identity map are below. Note that the equilibrium population is where the updating function and identity map intersect. Also included is the cobwebbing for the first few iterations, showing the solution heading toward the equilibrium.