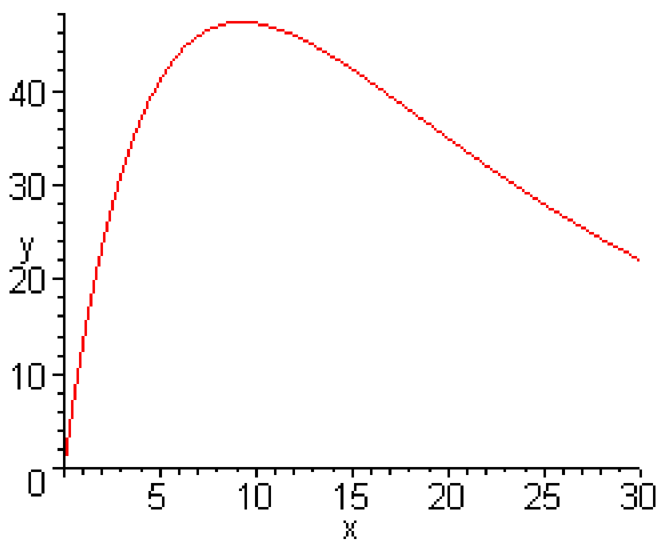
3. Rewrite the function $f(x) = 5\ln\left(\frac{1}{x}\right) - e^{-2x} + 2 = 5\ln(x^{-1}) - e^{-2x} + 2 = -5\ln(x) - e^{-2x} + 2$, then

$$f'(x) = -5\left(\frac{1}{x}\right) - (-2)e^{-2x} + 0 = -\frac{5}{x} + 2e^{-2x}.$$

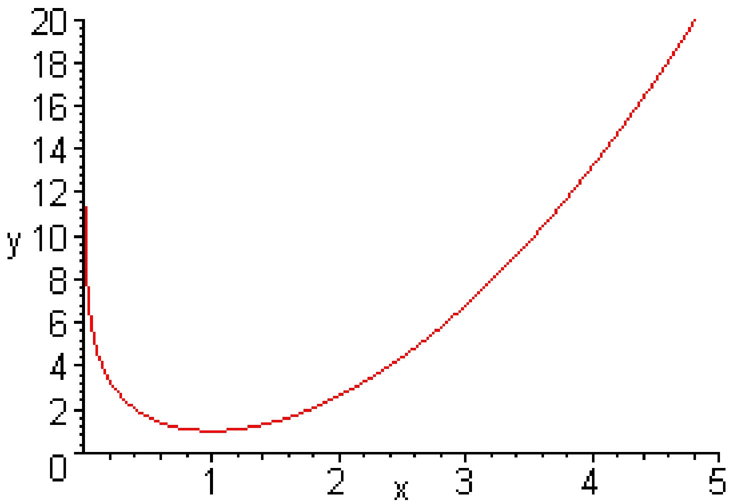
4. Rewrite the function
$f(x) = \frac{3}{e^{5x}} + 4\ln\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{x} = 3e^{-5x} + 4\ln(x^{-\frac{1}{2}}) - x^{-1} = 3e^{-5x} - 2\ln(x) - x^{-1}$, then

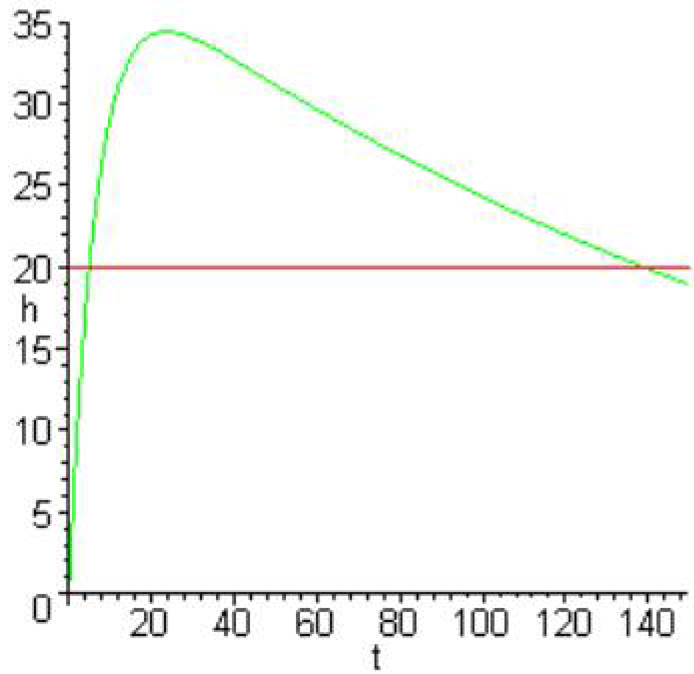
$$f'(x) = 3(-5)e^{-5x} - \frac{2}{x} - (-1)x^{-2} = -15e^{-5x} - \frac{2}{x} + \frac{1}{x^2}.$$

5. $y = 100\left(e^{-0.05x} - e^{-0.2x}\right)$,
Domain is all x.
y-intercept: $y(0) = 100(1-1) = 0$, so $(0,0)$.
x-intercept: Also, $(0,0)$
Horizontal asymptote: As $x \to \infty$, $y \to 0$.
Derivative $y'(x) = 100\left((-0.05)e^{-0.05x} + (0.2)e^{-0.2x}\right)$
Critical points when $y'(x) = 0$ or $0.05e^{-0.05x} = 0.2e^{-0.2x}$. Equivalently, $e^{-0.05x}e^{0.2x} = \frac{0.2}{0.05} = 4$, so $e^{0.15x} = 4$. Thus, $0.15x = \ln(4)$ or $x = \frac{20}{3}\ln(4) \simeq 9.242$. $y(9.242) = 47.25$, so there is a maximum at $(4.242, 47.25)$.



7. $y = x^2 - 2\ln(x)$,
Domain is $x > 0$.
y-intercept: Outside the domain, so none.
x-intercept: This equation cannot be solved for x. However, the minimum is positive, so none exist.
Vertical asymptotes. $x = 0$.
Derivative: $y'(x) = 2x - 2\left(\frac{1}{x}\right) = 2\left(\frac{x^2-1}{x}\right)$.
Critical points when $y'(x) = 0$ or $x^2 - 1 = 0$. Since $x > 0$, then $x = 1$.
$y(1) = 1$, so there is a minimum at $(1, 1)$.
Graph is below to the left.





9. a. Since $h(t) = 40\left(e^{-0.005t} - e^{-0.15t}\right)$, it follows that

$$h'(t) = 40\left(-0.005e^{-0.005t} - (-0.15)e^{(} - 0.15t)\right) = 40(0.15e^{(} - 0.15t) - 0.005e^{-0.005t}).$$

The maximum occurs when $h'(t) = 0$, so $0.15e^{(} - 0.15t) = 0.005e^{-0.005t}$ or $e^{-0.005t}e^{0.15t} = \frac{0.15}{0.005}$. Thus, $e^{(.15-.005)t} = e^{0.145t} = 30$ or $0.145t = \ln(30)$. The maximum is at $t_c = \frac{1}{0.145}\ln(30) \simeq$ 23.46 days. The maximum concentration is $h(t_c) = 40\left(e^{-0.005t_c} - e^{-0.15t_c}\right) = 34.39$ ng/dl.

b. The only intercept is $(0, 0)$. There is a horizontal asymptote at $h = 0$, since $\lim_{t\to\infty} h(t) = 0$. Maple can be used to show the $h(t) = 20$ at $t = 5.0$ and 138.6, so the hormone level remains above 20 ng/dl of blood for about 134 days. The graph is shown above to the right.

11. a. Let the population be described by $P(t) = 3.93e^{at}$. With the population in 1800 being 5.31 million, $5.31 = 3.93e^{10a}$, so $10a = \ln\left(\frac{5.31}{3.93}\right) \simeq= 0.301$ or $a = 0.0301$.

b. The derivative of this function is $P'(t) = 3.93ae^{at} = 0.1183e^{0.0301t}$, which gives the annual rate of growth (in millions/yr).

c. In 1850, $P(60) = 3.93e^{0.0301(60)} = 23.91$ million with a growth rate of $P'(60) = 0.1183e^{0.0301(60)} = 0.720$ million/yr. In 1860, $P(70) = 3.93e^{0.0301(70)} = 32.31$ million with a growth rate of $P'(70) = 0.1183e^{0.0301(70)} = 0.972$ million/yr.

d. The percent errors in 1850 and 1860 are

$$\begin{aligned} 100\frac{(23.9 - 23.2)}{23.2} &= 3.0\% \\ 100\frac{(32.3 - 31.4)}{31.4} &= 2.9\% \end{aligned}$$

e. The annual growth rate is $\frac{(31.4-23.2)}{10} = 0.82$ million/yr, while the average of the growth rates in Part c is $\frac{0.720+0.972}{2} = 0.846$ million/yr, which is very close.

14. a. Since $E(x) = 0.774 + 0.727\ln(x)$, it follows that

$$E'(x) = \frac{0.727}{x}.$$

b. $E(10,000) = 0.774 + 0.727\ln(10,000) = 7.47$ kJ/day, while $E'(10,000) = \frac{0.727}{10{,}000} = 7.27 \times 10^{-5}$ kJ/day/g. Biologically, the first result says that a 10 kg pronghorn fawn burns about 7.47 kJ/day in addition to the energy put into growth. The second result states that each gram of growth adds an additional 7.27×10^{-5} kJ/day of energy expended when the fawn is near 10 kg in weight.

c. Below is the graph.