Find the derivatives of the following functions:

1. $f(x) = x^2 - 3e^{-x} - 1$,

2. $f(x) = 2x - 7\ln(x) + e^{2x}$,

3. $f(x) = 5\ln\left(\frac{1}{x}\right) - e^{-2x} + 2$,

4. $f(x) = \frac{3}{e^{5x}} + 4\ln\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{x}$.

Sketch the curves of the functions below. Give the domain of each of the functions. List all maxima and minima for each graph. Also, give the x and y-intercepts and any asymptotes if they exist.

5. $y = 100\left(e^{-0.05x} - e^{-0.2x}\right)$,

6. $y = 20\left(1 - e^{-x}\right)$,

7. $y = x^2 - 2\ln(x)$,

8. $y = 4\ln(x)$,

9. Some hormones have a strong effect on mood, so finding a delivery device that delivers a hormone at a more constant level over a longer period of time is important for hormone therapy. Suppose that a drug company finds a polymer that can be implanted to deliver a hormone, $h(t)$, which is experimentally found to satisfy

$$h(t) = 40\left(e^{-0.005t} - e^{-0.15t}\right),$$

where h is in nanograms per deciliter of blood (ng/dl) and t is in days.

a. Find the maximum concentration of this hormone in the body and when this occurs.

b. Determine all intercepts and asymptotes, then graph $h(t)$ for $0 \leq t \leq 150$. Use the graph to approximate how long the hormone level remains above 20 ng/dl.

10. Let $Y(t)$ be a population of yeast in a sugar solution that begins with a concentration of 10 yeast/ml.

a. If the concentration of yeast is given by

$$Y(t) = 10e^{at},$$

then find the value of a assuming that the concentration doubles every 2 hours.

b. Differentiate this function to find the rate of increase in the concentration of yeast per hour.

c. Evaluate the concentration of yeast at $t = 1$, 2, and 5 hours, and the rate of increase in the concentration of yeast per hour.

11. In an earlier section, we studied the population of the U. S. The population in 1790 was 3.93 million, and the growth rate was about 35% per decade.

a. If the population $P(t)$ is increasing exponentially, then the population at time t can be described by

$$P(t) = 3.93e^{at},$$

where P is in millions and t is in years after 1790. The population in 1800 is 5.31 million. , Determine a in the expression above.

b. Differentiate this function to find the function which represents the annual rate of growth (in millions/yr).

c. Use the expressions in Parts a. and b. to estimate the population in 1850 and 1860 and the annual growth rates at each of those dates.

d. If the actual populations in 1850 and 1860 are 23.2 and 31.4 million, respectively, then determine the percent error between this model and the actual populations.

e. Take the difference of the populations in 1850 and 1860 and divide by 10 to estimate the annual growth rate for that decade and compare that value to the values you obtained in Part c.

12. White lead, ^{210}Pb, is a radioactive element that appears in the pigment of paints and can be used to date oil paintings. This helps determine modern art forgeries. ^{210}Pb undergoes a β- decay to ^{210}Bi. Radioactive substances decay at a rate proportional to the amount of the substance available.

a. Suppose that a 1 g sample of paint contains 6 μg of ^{210}Pb. The amount of ^{210}Pb, $R(t)$ satisfies the equation,

$$R(t) = 6e^{-kt},$$

where the constant k is to be determined. If the half-life of ^{210}Pb is 22 years, then find k.

b. Find $R'(t)$, then determine the rate of change in the amount of ^{210}Pb at $t = 20$, 50, and 100 years.

c. Suppose a fresh 1 g sample of pigment gives 60 counts per minute (cpm) (from the β decay of the ^{210}Pb), and a 1 g sample of the same pigment from a historic painting releases 8 cpm, estimate the age of the painting.

13. The cutlassfish is a valuable resource in the marine fishing industry in China. A von Bertalanffy model is fit to data for one species of this fish giving the length of the fish, $L(t)$ (in mm), as a function of the age, a (in yr). An estimate of the length of this fish is

$$L(a) = 589 - 375e^{-0.168a}.$$

a. Find the L-intercept and any asymptotes. What is the maximum possible length of this fish?

b. Determine how long it takes for this fish to reach 90% of its maximum length. Sketch a graph of the von Bertalanffy model.

c. Differentiate $L(a)$ with respect to a, then determine how fast the average fish is growing when it is 5 years old.

14. The log of the field metabolic rate (FMR) or the log of the total energy expenditure per day in excess of growth is calculated for pronghorm fawns using Nagy's formula

$$E(x) = 0.774 + 0.727\ln(x),$$

where x is the mass of the fawn (in g) and $E(x)$ is the log of the energy expenditure (in kJ/day).

a. Compute the derivative $E'(x)$.

b. Find the log of the energy expenditure when $x = 10,000$, then compute $E'(10,000)$. Give a biological interpretation of these results.

c. Graph $E(x)$ for $x \in [5000, 20000]$.

15. a. It has been shown that iron is the primary limiting nutrient in open ocean waters. There are currently a number of experiments to see if seeding the ocean with iron can create an algal bloom that fixes CO_2 (to remove this greenhouse gas). Soluable iron that is dumped into the ocean is rapidly used by algae, which are consumed by other organisms. At $t = 0$, a research

vessel from Scripps Institute of Oceanography dumps 500 kg of soluable iron. Measurements from a trailing ship indicate that the amount of iron remaining in the water (not in the algae) satisfies the equation:

$$F(t) = 500\, e^{-0.23t},$$

where t is in days. Find how long it takes for the amount of soluable iron to reach the level of 100 kg remaining. Sketch a graph of F showing the F-intercept and the horizontal asymptote.

b. Find the derivative $\frac{dF}{dt}$. Determine the rate of change of soluable iron at $t = 2$.

c. As noted above the algae rapidly blooms, then fades as the iron passes to organisms higher in the food web. Suppose that samples of the sea water give a population of algae, $P(t)$, (in thousands/cc) satisfying the following equation:

$$P(t) = 10\left(e^{-0.05t} - e^{-0.8t}\right),$$

where t is in days. Find the derivative $\frac{dP}{dt}$. Find when the algal population achieves its maximum concentration and determine what its maximum concentration is. Sketch a graph of P showing the P-intercept, the maximum, and any horizontal asymptotes.