2. The function $f(x) = x^2(x^3 - 2x + 1)^3$ is the product of two functions with the second requiring the chain rule. The derivative of $f_2(x) = (x^3 - 2x + 1)^3$ is $f_2'(x) = 3(x^3 - 2x + 1)^2(3x^2 - 2)$. By the product rule,

$$f'(x) = x^2\left[3(x^3 - 2x + 1)^2(3x^2 - 2)\right] + 2x(x^3 - 2x + 1)^3.$$

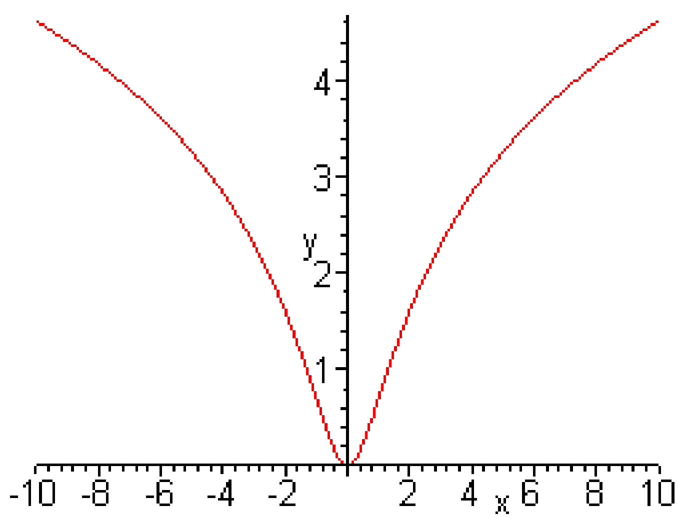
4. The function $f(x) = (x^2 - e^{-x^2})^3$ is the composition of the functions $g(u) = u^3$ and $h(x) = x^2 - e^{-x^2}$. The derivative of $h(x)$ uses the chain rule with $h'(x) = 2x - e^{-x^2}(-2x)$, since $\frac{d(e^{k(x)})}{dx} = k'(x)e^{k(x)}$. Also, $g'(u) = 3u^2$. Combining these results with $u = x^2 - e^{-x^2}$ gives

$$f'(x) = 3(x^2 - e^{-x^2})^2(2x + 2xe^{-x^2}).$$

6. For $y = \ln(x^2 + 1)$, the derivative is $y' = \dfrac{2x}{x^2 + 1}$, since by the chain rule, $\frac{d(\ln(g(x)))}{dx} = \frac{g'(x)}{g(x)}$.
This is an even function.
The only intercept is $(0, 0)$.
There are no asymptotes, since the argument is strictly greater than zero.
Critical points occur when $y' = 0$, so the numerator $2x = 0$ or $x = 0$.
There is a minimum at $(0, 0)$.
The second derivative uses the quotient rule, giving

$$y'' = \frac{2(x^2 + 1) - 2x(2x)}{(x^2 + 1)^2} = \frac{2(1 - x^2)}{(x^2 + 1)^2}.$$

Thus, there are points of inflection at $(\pm 1, \ln(2)) \simeq (\pm 1, 0.693)$.
The graph is below.



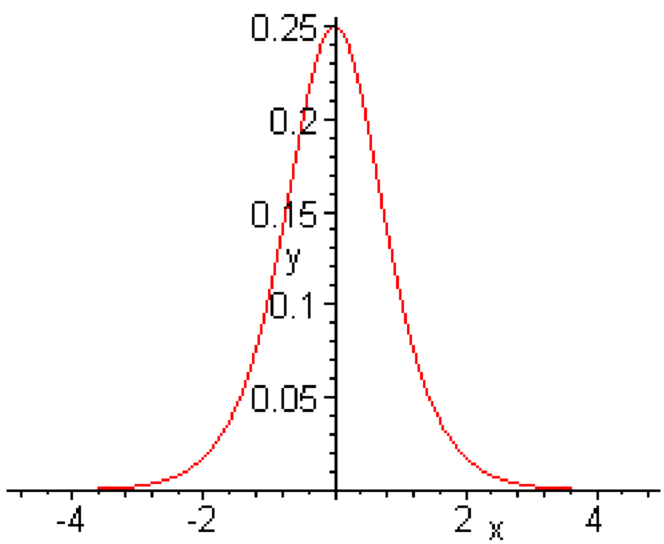
8. The derivative combines the quotient rule and chain rule and satisfies

$$\begin{aligned} y' &= \frac{2e^{2x}(1+e^{2x})^2 - e^{2x}(2e^{2x}(1+e^{2x}))}{(1+e^{2x})^4} \\ &= \frac{2e^{2x}(1-e^{2x})}{(1+e^{2x})^3.} \end{aligned}$$

Since $y(-x) = y(x)$ (after some algebra), this is an even function. Critical points are found by solving $y'(t) = 0$, which occurs when the numerator is zero above or $1 - e^{2x} = 0$, but this only occurs at $x = 0$. It follows that there is a maximum at $(0, 0.25)$. Since $y(x)$ is always positive, there is only a y-intercept at $(0, 0.25)$. Since the denominator has a e^{4x}, which is larger than the e^{2x} in the numerator, the denominator goes to infinity faster, which implies a horizontal asymptote at $y = 0$. The second derivative, using the quotient and chain rules, is given by

$$\begin{aligned} y'' &= \frac{(1+e^{2x})^3(4e^{2x} - 8e^{4x}) - 2(e^{2x} - e^{4x})6(1+e^{2x})^2 e^{2x}}{(1+e^{2x})^6} \\ &= \frac{4e^{2x}(e^{4x} - 4e^{2x} + 1)}{(1+e^{2x})^4}. \end{aligned}$$

Thus, the points of inflection occur when $e^{4x} - 4e^{2x} + 1 = 0$. Let $z = e^{2x}$, then this equation is $z^2 - 4z + 1 = 0$, which has roots when $z = 2 \pm \sqrt{3}$. Solving $e^{2x} = 2 + \sqrt{3}$ gives $x = \frac{\ln(2+\sqrt{3})}{2} = 0.6585$, and similarly, $e^{2x} = 2 - \sqrt{3}$ gives $x = \frac{\ln(2-\sqrt{3})}{2} = -0.6585$. It follows that the points of inflection are $(\pm 0.6585, 0.1667)$. Graph is below.



9. a. Rate of growth in height is by the derivative of the height equation, so the rate of growth is $h'(a) = 6.44$ cm/yr.

b. Weight as a function of age $W(a) = 0.0000302(6.44a + 82.1)^{2.84}$. The derivative is $W'(a) = 0.0000302(2.84)(6.44a + 82.1)^{1.84} = 0.0005523(6.44a + 82.1)^{1.84}$ kg/yr.

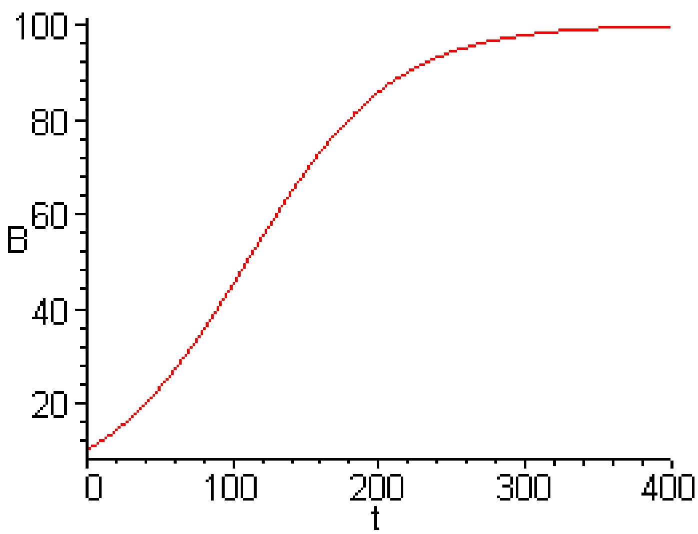
c. Rate of change of weight at ages 4, 8, and 13 is found by substituting into the expression above for the derivative giving: $W'(4) = 3.039$ kg/yr, $W'(8) = 4.506$ kg/yr, and $W'(13) = 6.704$ kg/yr.

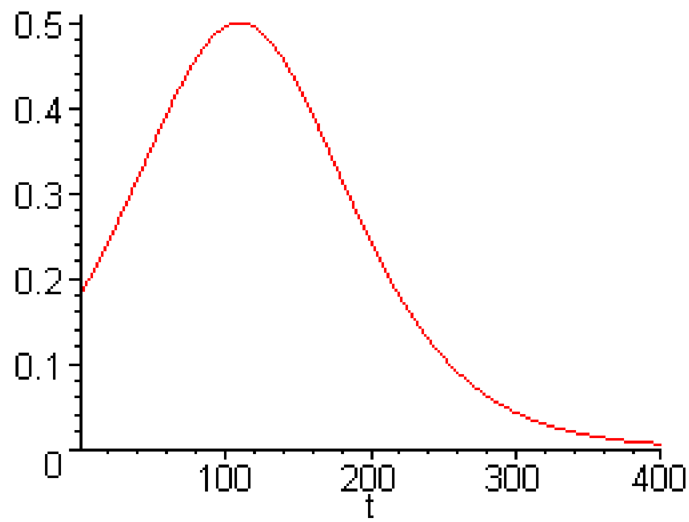
11. a. If we write $B(t) = 100(1+9e^{-0.02t})^{-1}$, then the chain rule gives the derivative $B'(t) = -100(1+9e^{-0.02t})^{-2}(-0.18e^{-0.02t}) = \dfrac{18e^{-0.02t}}{(1+9e^{-0.02t})^2}$. The second derivative is found by combining the quotient and chain rules to the expression for $B'(t)$ giving

$$\begin{aligned} B''(t) &= \frac{(1+9e^{-0.02t})^2(-0.36e^{-0.02t}) - 18e^{-0.02t}(2(1+9e^{-0.02t})(-0.18e^{-0.02t}))}{(1+9e^{-0.02t})^4} \\ &= \frac{0.36(9e^{-0.04t} - e^{-0.02t})}{(1+9e^{-0.02t})^3} = \frac{0.36e^{-0.02t}(9e^{-0.02t} - 1)}{(1+9e^{-0.02t})^3}. \end{aligned}$$

b. Since $B(0) = 100/(1+9) = 10$, the B-intercept is $(0, 10)$. As $t \to \infty$, $e^{-0.02t} \to 0$, so there is a horizontal asymptote at $B = 100$. The point of inflection is found by solving $9e^{-0.02t} - 1 = 0$, which gives $e^{0.02t} = 9$ or $t = 50\ln(9) = 109.9$. Thus, the point of inflection is $(109.9, 50)$. Below is a sketch of the graph to the left.

c. $B'(0) = 0.18$. There is a horizontal asymptote $B' = 0$. The maximum of the derivative occurs at $(109.9, 0.5)$, where the second derivative is zero. Below is a sketch of the graph to the right.





13. a. Since $L(0) = 0$, the intercept is $(0, 0)$. Since $e^{-0.4t} \to 0$ as $t \to \infty$, there is a horizontal asymptote at $L = 16$ cm. The graph is shown below.

b. The composite function is given by $W(t) = 0.07(16)^3(1 - e^{-0.4t})^3 = 286.72(1 - e^{-0.4t})^3$. By similar arguments to Part a., this has an intercept $(0, 0)$, and the horizontal asymptote is $W = 286.72$ g. The graph is shown below.

c. By the chain rule, the derivative of $W(t)$ is $W'(t) = 286.72(3(1 - e^{-0.4t})^2(0.4)e^{-0.4t}) = 344.064e^{-0.4t}(1 - e^{-0.4t})^2$. With the product and chain rules, the second derivative of $W(t)$ is $W''(t) = 344.064(e^{-0.4t}2(1 - e^{-0.4t})(0.4)e^{-0.4t} - 0.4e^{-0.4t}(1 - e^{-0.4t})^2) = 137.6256e^{-0.4t}(1 - e^{-0.4t})(3e^{-0.4t} - 1)$. The weight of the sculpin is increasing most rapidly at the point of inflection, which occurs when $W''(t) = 0$ or $(1 - e^{-0.4t})(3e^{-0.4t} - 1) = 0$. The first factor is zero when $t = 0$ (which is when $W'(t) = 0$, so no weight increase), while the second factor is zero when $e^{0.4t} = 3$ or $t = \frac{5}{2}\ln(3) \simeq 2.75$ yrs. The average sculpin weighs 84.95 g at this age and is increasing in weight at a rate of 50.97 g/yr.