Find the derivatives of the following functions:

1. $f(x) = (x^2 - 3x + 4)^4$,

2. $f(x) = x^2(x^3 - 2x + 1)^3$,

3. $f(x) = \dfrac{e^{x^2}}{2x+1} + \ln(x^2)$,

4. $f(x) = (x^2 - e^{-x^2})^3$.

Find the derivative and sketch the curves of the functions below. Are these functions even, odd, or neither? List all maxima and minima for each graph. Find the second derivative of these functions, then locate the points of inflection. Also, give the $x$ and $y$-intercepts and any asymptotes if they exist.

5. $y = 2e^{-x^2/2}$,

6. $y = \ln(x^2 + 1)$,

7. $y = \dfrac{10x}{(1 + 0.1x)^2}$,

8. $y = \dfrac{e^{2x}}{(1 + e^{2x})^2}$,

9. A study of American girls ages 4-13 in the $90^{th}$ percentile found that their height $h$ (in cm) as a function of their age $a$ (in years) satisfies the equation

$$h(a) = 6.44a + 82.1.$$

The same study found that their weight $W$ (in kg) as a function of their height is given by

$$W(h) = 0.0000302h^{2.84}.$$

a. What is a rate of growth in height? Be sure to include units in your answer.

b. Write an expression for the composite function that gives the weight as a function of age. Differentiate this function to find $W'(a)$ using the chain rule.

c. What is the rate of change in weight at ages 4, 8, and 13? Be sure to include units for your answer.

10. Hassell's model is often used to study populations of insects. Suppose that the updating function for the population of a species of moth $P$ is given by

$$H(P) = \frac{5P}{(1+0.002P)^4}.$$

a. Find all equilibria of the model by solving the equation $H(P_e) = P_e$.

b. Determine the intercepts, all extrema, and any asymptotes for $P \geq 0$, then sketch a graph of $H(P)$.

11. The continuous logistic growth model is a very important model used in Biology. Suppose that a population of bacteria satisfies the logistic growth model

$$B(t) = \frac{100}{1+9e^{-0.02t}},$$

where $t$ is in minutes and $B$ is in thousands of bacteria/ml.

a. Compute both the first and second derivatives of $B(t)$.

b. Find the $B$-intercept and any asymptotes for the model of this population, then sketch a graph of $B(t)$. Also, find the point of inflection.

c. Determine $B'(0)$ and find any asymptotes for the function $B'(t)$. Find the maximum of this function, then sketch its graph.

12. The lecture notes have an example for the weight of a lake trout as a function of age, and it was given by the formula

$$W(t) = 25.92(1-e^{-0.2t})^3,$$

where $W$ is in kg and $t$ is in years. Find the age at which the lake trout are increasing their weight most rapidly.

13. The growth in length of sculpin is approximated by the von Bertalanffy equation

$$L(t) = 16(1 - e^{-0.4t}),$$

where $t$ is in years and $L$ is in cm. An allometric measurement of sculpin shows that their weight can be approximated by the model

$$W(L) = 0.07L^3,$$

where $W$ is in g.

a. Find the intercepts and any asymptotes for the length of a sculpin, then sketch of graph showing the length of a sculpin as it ages.

b. Create a composite function to give the weight of the sculpin as a function of its age, $W(t)$. Find the intercepts and any asymptotes for $W(t)$, then sketch of graph showing the weight of a sculpin as it ages.

c. Find the derivative of $W(t)$ using the chain rule. Also, compute the second derivative, then determine when this second derivative is zero. From this information, find at what age the sculpin are increasing their weight the most and determine what that weight gain is. Be sure to give the units of weight gain.

14. Suppose that after a burn a pioneering plant community has its biomass accumulating according to the following growth model,

$$P(t) = 20(1 - e^{-0.2t}),$$

where $t$ is in years and $P$ is in metric tons. The herbivores that graze on this plant community satisfy the equation

$$H(P) = 3(1 - e^{-0.1P}),$$

where $H$ is in metric tons of the biomass of herbivores.

a. Sketch a graph of $P(t)$, showing any intercepts and asymptotes. Compute the derivative to determine the rate of change in biomass of the plant material. What is the rate of change in biomass at $t = 0$, 2, 10, and 20 years?

b. Create the composite function to find the biomass of the herbivores as a function of time $H(t)$. Differentiate this function, then find the rate of change in biomass of the herbivores at $t = 0$, 2, 10, and 20 years