

1. Suppose that $e^a = 3.7$ and $e^b = 0.4$. In addition, assume that $\ln(c) = -1.5$ and $\ln(d) = 2.1$. Use the properties of exponentials and logarithms to evaluate the following:

a. $\frac{(e^0 + e^a)^2}{e^{a-b}},$

b. $\frac{\ln(d^2/c) - \ln(e)}{(\ln(cd) + \ln(1))}.$

2. Suppose that $e^a = 2.4$ and $e^b = 0.6$. In addition, assume that $\ln(c) = -2.6$ and $\ln(d) = 3.1$. Use the properties of exponentials and logarithms to evaluate the following:

a. $\frac{e^0 + e^{2b}}{e^{a-b}},$

b. $\frac{\ln(d^2/c) + \ln(e)}{\ln(1) - \ln(c^3)}.$

Graph the following exponentials. Determine all x and y -intercepts for these functions and find any horizontal asymptotes.

3. $f(x) = e^{-x} - 2$

4. $f(x) = 2 + e^{2x}$

5. $f(x) = 10 - e^{-x/2}$

6. $f(x) = e^x - 1$

Find the domain for the following functions and graph the logarithms. Determine all x and y -intercepts for these functions and find any vertical asymptotes.

7. $f(x) = \ln(2x)$

8. $f(x) = 3 + \ln(x)$

9. $f(x) = \ln(4 - x)$

10. $f(x) = 2 - \ln(1 - x)$

11. Research has shown that the average number of mammalian species N on an island satisfies the equation

$$N = kA^{\frac{1}{3}}$$

where A is the area (in km^2) of the island and $k = 2$.

- a. Find the expected number of mammals on islands with 125 and 8000 km^2 .
- b. If you discovered an island had 32 different species of mammals, then, based on the formula above, approximately how large is the island?
- c. Sketch a graph of the number of mammalian species on an island vs. the area of the island. Plot the points found in Parts a and b.

12. The Crew Classic rowing event on Mission Bay is held each year in spring. It can be shown that the times, t , of a particular race satisfy a power law with respect to the number of men, n , in the boat,

$$t = kn^a.$$

You are given that the winning time for the eight man crew was exactly 6 min., while the winning time for the four man crew was 6 min 28.8 sec (Remember to convert the seconds to decimal minutes.)

- a. With the information given above find the value for k and a .
- b. Use your answer from part a to determine likely winning times for the pairs (2 oarsmen) and singles (1 oarsman). List one or two problems with the model for predicting the winning times for this event.

13. The power generated by a windmill satisfies an allometric model

$$P = kv^a,$$

where P is the power in watts and v is the velocity of the wind. Experiments have determined that when the wind velocity is 8 mph the windmill generates 0.467 watts. It generates 1.643 watts when the wind is 15 mph.

- a. Find the proportionality constant k and the power a .
- b. Find how many watts are generated in a wind of 13 mph. Also, determine the wind velocity necessary to generate 2 watts.

14. The class notes use an allometric model to relate the weight of an animal to its pulse, given by $P = kw^a$, where P is the pulse and w is the weight.

a. You are given that a hummingbird weighs 4 grams and has a pulse of 615 beats/min and a sparrow weighs 28 grams and has a pulse of 350. Find the constants k and a in the allometric model using these data.

b. From the model you produced in Part a., estimate the pulse of an 11 gram wren and the weight of a dove that has a pulse of 130 beats/min.

15. Data suggest that the lifetime of erythrocytes (red blood cells) for mammals satisfy an allometric model. The average lifetime for erythrocytes in a 70 kg man is 120 days. The average lifetime for erythrocytes in a 1.5 kg rabbit is 65 days. Use these data to find an allometric model for the lifetime of erythrocytes as a function of weight, *i.e.*,

$$T = kw^a.$$

Find the constants k and a . Use this model to determine the average lifetime for erythrocytes in a 20 kg dog. Also, determine the weight of an animal whose erythrocytes live for 100 days.

16. In Gulliver's Travels, the Lilliputians decided to feed Gulliver 1728 times as much food as a Lilliputian ate. They reasoned that, since Gulliver was 12 times their height, his volume was $12^3 = 1728$ times the volume of a Lilliputian and so he required 1728 times the amount of food one of them ate. Why was their reasoning wrong? What is the correct answer?