1. a. $f'(x) = 18x^2 - \frac{4}{x^3} - 2e^{2x}(x^2 + x - 9)$

b. $g'(x) = \frac{2}{x} - 6e^{-3x}$

c. $h'(x) = 12x^5 \ln(x) + 2x^5 + \frac{9}{x^4} - 2e^{-4x}$

d. $k'(t) = \frac{t}{2} + \frac{2}{\sqrt{t^3}} + \frac{1}{t}$

e. $p'(w) = 6w^5 - 6w^2 - \frac{2}{5}w^{-3/5} - \frac{e^{-w}}{w^3} \left(1 + \frac{3}{w}\right)$

f. $q'(z) = 2A(1 + \ln(z)) - \frac{B}{2\sqrt{z}} - \frac{3C}{z^4}$

2. a. $y = 27x - x^3$
   Domain is all $x$.
   $y$-intercept: $(0, 0)$.
   $x$-intercepts: $x = 0, \pm 3\sqrt{3}$.
   No asymptotes
   Derivative: $y'(x) = 27 - 3x^2$
   Maximum: $(3, 54)$, and Minimum: $(3, -54)$.
   Second derivative: $y''(x) = -6x$.
   Point of inflection: $(0, 0)$.

b. $y = x^4 - 4x$
   Domain is all $x$.
   $y$-intercept: $(0, 0)$.
   $x$-intercepts: $x = 0, 3\sqrt{4} \approx 1.587$.
   No asymptotes
   Derivative: $y'(x) = 4x^3 - 4$
   Minimum: $(1, -3)$
   Second derivative: $y''(x) = 12x^2$.
   Point of inflection: $(0, 0)$. 
c. \(y = x^3 + 3x^2 + 3x + 1\)
Domain is all \(x\).
y-intercept: \((0, 1)\).
x-intercept: \((-1, 0)\).
No asymptotes
Derivative: \(y'(x) = 3x^2 + 6x + 3\)
Critical point at \(x = -1\) with \((-1, 0)\) a saddle point.
Second derivative: \(y''(x) = 6x + 6\).
Point of inflection: \((-1, 0)\).

d. \(y = 18x^2 - x^4\)
Domain is all \(x\).
y-intercept: \((0, 0)\).
x-intercept: \(x = 0, \pm 3\sqrt{2}\).
No asymptotes
Derivative: \(y'(x) = 36x - 4x^3\)
Critical points: \(x = 0, \pm 3\), with \((0, 0)\) is a minimum and local maxima at \((-3, 81)\) and \((3, 81)\).
Second derivative: \(y''(x) = 36 - 12x^2\).
Points of inflection: \(\pm 3, 45\).  

e. \(y = x + \frac{4}{x} = x + 4x^{-1}\)
Domain is all \(x \neq 0\).
Vertical asymptote at \(x = 0\).
No \(x\) or \(y\)-intercepts.
Derivative: \(y'(x) = 1 - 4x^{-2}\).
Critical points: \(x = \pm 2\).
\((-2, -4)\) is a local maximum and \((2, 4)\) is a local minimum.
Second derivative: \(y''(x) = 8x^{-3}\)
No points of inflection.
f. \( y = 4xe^{-0.02x} \)
Domain is all \( x \).
\( x \) and \( y \)-intercept: \((0, 0)\).
Horizontal asymptote: \( y = 0 \) (to the right).
Derivative: \( y'(x) = 4e^{-0.02x}(1 - 0.02x) \)
Critical point: \( x = 50 \). \((50, 73.576)\), a maximum.
Second derivative \( y''(x) = -0.16(1 - 0.01x)e^{-0.02x} \).
Point of inflection: \((100, 54.134)\).

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g. \( y = x \ln(x) \)
Domain is \( x > 0 \).
No \( y \)-intercept.
\( x \)-intercept: \( x = 1 \).
No asymptotes. (As \( x \to 0 \), \( y \to 0 \).)
Derivative: \( y'(x) = 1 + \ln x \).
Critical point: \( x = e^{-1} \simeq 0.3679 \). Minimum: \((e^{-1}, -e^{-1})\).
Second derivative \( y''(x) = \frac{1}{x} \). No point of inflection.

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h. \( y = (x - 4)e^{2x} \)
Domain is all \( x \).
\( y \)-intercept: \((0, -4)\).
\( x \)-intercept: \( x = 4 \).
Horizontal asymptote: \( y = 0 \) (to the left).
Derivative: \( y'(x) = (2x - 7)e^{2x} \).
Critical point: \((3.5, -0.5e^{7}) \simeq (3.5, -548.3)\), a minimum.
Second derivative \( y''(x) = 4(x - 3)e^{2x} \).
Point of inflection: At \((3, -e^{6}) \simeq (3, -402.4)\).
3. a. The average velocity over the first two seconds is 16 ft/sec.

   b. The velocity is $v(t) = 48 - 32t$. The velocity is zero at $t = 1.5$ sec. The maximum height of the ball is $h_{\text{max}} = 196$ ft. At $t = 1$ sec, the velocity is $v(1) = 16$ ft/sec.

   c. The ball hits the ground at 5 sec with a velocity of $v(5) = -112$ ft/sec. The graph is below.

4. a. The velocity is $v(t) = h'(t) = v_0 - 32t$.

   b. The velocity is zero when $t = \frac{v_0}{32}$ sec. The antelopes have to launch themselves at 16 ft/sec to clear a 4 ft fence.

   c. The antelopes stay in the air for one second. They hit the ground with a velocity of $v(1) = -16$ ft/sec.
5. a. Asymptotically, the leopard shark can reach 2.1 m. The length of the leopard shark at birth is 0.2 m, at 1 yr is 0.62 m, at 5 yr is 1.56 m, and at 10 yr is 1.94 m. The maximum length is 2.1 m. The shark reaches 90% of its maximum length at $t = 8.81$ yr. The graph is below.

b. The rate of change of body length per year is

$$\frac{dL}{dt} = 0.475e^{-0.25t}.$$  

The annual growth rate at birth is 0.475 m/yr, at 1 yr is 0.370 m/yr, at 5 yr is 0.136 m/yr, and at 10 yr is 0.0390 m/yr.
6. a. The populations $P_1 = 2300$ and $P_2 = 3795$.

b. The equilibria are $P_e = 0$ and $P_e = 3600$. The derivative of the updating function is $F'(P) = 2.8 - 0.001P$. At $P_e = 0$, $F'(0) = 2.8 > 1$, so this equilibrium is unstable with solutions monotonically growing away from $P_e = 0$. At $P_e = 3600$, $F'(3600) = -0.8 > -1$, so $P_e = 3600$ is stable with solutions oscillating, but approaching this equilibrium.

c. The updating function has $P$-intercepts at $P = 0$ and $P = 5600$. The vertex is $(2800, 3920)$. The updating function intersects the identity function at the equilibria, $(0, 0)$ and $(3600, 3600)$. The graph is shown below.
7. a. The populations are \( P_1 = 982 \) and \( P_2 = 2207 \).

   b. The derivative of \( R(P) \) is \( R'(P) = 6(1 - .001P)e^{-0.001P} \). The maximum of \( R(P) \) occurs at \( P = 1000 \) with \( R(1000) = 6000e^{-1} = 2207 \). As \( P \to \infty \), the exponential dominates the polynomial part, so \( R(P) \to 0 \). The graph of the Ricker’s function is below.

   c. The equilibria are \( P_e = 0 \) and \( P_e = 1000 \ln(6) = 1792 \). At \( P_e = 0 \), \( R'(0) = 6 > 1 \), so this equilibrium is unstable with solutions monotonically growing away from \( P_e = 0 \). At \( P_e = 1792 \), \( R'(1792) = -0.792 > -1 \), so the higher equilibrium is stable with solutions oscillating, but approaching \( P_e = 1792 \).
8. a. The populations are $P_1 = 1,061,845$, $P_2 = 1,126,877$, and $P_3 = 1,195,223$.

b. The derivative of $G(P)$ is $G'(P) = 1.19 - 0.01 \ln(P)$. The maximum occurs at $P_{max} = e^{119} = 4.80 \times 10^{51}$ with $G(P_{max}) = 0.01e^{119} = 4.80 \times 10^{49}$. The $P$-intercept is $P = e^{120} = 1.30 \times 10^{52}$. The graph is shown below.

c. The equilibrium is $P_e = e^{20} = 485 \times 10^6$. At $P_e = e^{20}$, $G'(e^{20}) = 0.99 < 1$, so the equilibrium is stable with solutions monotonically approaching $P_e = e^{20}$.
9. a. The concentration of glucose reaches 90 mg/100 ml of blood in about $t = 2.15$ hours. The glucose function has a $g$-intercept of 160 and a horizontal asymptote of $g = 70$. The graph for the concentration of glucose in the blood is below.

b. The rate of change of glucose per hour is

$$\frac{dg}{dt} = -63e^{-0.7t}.$$  

At $t = 1$, $g'(1) = -63e^{-0.7} = -31.28$ mg/100 ml of blood/hour.

c. The level of insulin achieves a maximum at $t = 10 \ln \left( \frac{5}{4} \right) = 2.23$ hr with a maximum concentration of $i(2.23) = 0.819$. This graph starts at $(0,0)$ and asymptotically approaches zero for large time. A graph of the insulin concentration is below also.

d. The rate of change of insulin per hour is $i'(t) = 5e^{-0.5t} - 4e^{-0.4t}$. The rate of change at $t = 1$ is $i'(1) = 0.351$ units/hour.
10. The approximate age of the painting is 95 years old.

11. a. The doubling time is \( t = 100 \ln(2) = 69.3 \) min.

   b. The kinetic constant \( k \) for the mutant colony is \( k = \frac{\ln(2)}{25} = 0.02773 \). The mutant colony is 20\% of the population of the colony at \( t = \frac{\ln(250)}{k-0.01} = 311.5 \) min.

   c. The original population at \( t = 500 \) is \( P(500) = 1000e^5 = 148,413 \) bacteria. The mutant population is \( M(500) = e^{500k} = 1,048,576 \) bacteria. The rate of growth of the original population is \( P'(500) = 10e^5 = 1,484 \) bacteria/min. The rate of growth of the mutant colony is \( M'(500) = ke^{500k} = 0.02772e^{3.86} = 29,073 \) bacteria/min.

12. a. If \( f(x) = \sqrt{9-3x} \), then

   \[ f'(x) = -\frac{3}{2\sqrt{9-3x}}. \]

   b. If \( f(x) = \frac{x}{x+2} \), then

   \[ f'(x) = \frac{2}{(x+2)^2}. \]

13. The domain of this function is \(-4 \leq x < 0 \) and \( 0 < x \leq 4 \). The function is undefined at \( x = 0 \), but for the other integers, \( f(-3) = 8, f(-2) = 1, f(-1) = 2, f(1) = 5, f(2) = 1, \) and \( f(3) = 1. \) The limit fails to exist at \( x = -2 \) and \( x = 0 \).

   \[ \lim_{x \to -3} f(x) = 8 \]
   \[ \lim_{x \to -1} f(x) = 2 \]
   \[ \lim_{x \to 1} f(x) = 2 \]
   \[ \lim_{x \to 2} f(x) = 1 \]
   \[ \lim_{x \to 3} f(x) = 6 \]

   This function is continuous at all values of \( x \) with \( -4 < x < 4 \), except \( x = -2, 0, 1, \) and \( 3. \)