1. a. \( H_1 = 2040 \) and \( H_2 = 2080.8 \). In the general solution is given by \( H_n = (1.02)^n H_0 = 2000(1.02)^n \).

b. The general solution is given by \( G_n = (1.03)^n G_0 = 200(1.03)^n \). It takes 23.5 generations for the population to double.

c. The populations are equal in 236 generations.

2. a. The annual growth rate is \( r = 0.011753 \) (about 1.2% per year) and the general equation is \( P_n = (1.011753)^n 179.3 \).

b. The model predicts a population of 286.1 million in the year 2000. The error between this and the actual population is 1.7%.

c. The population will double about 59.3 years after 1960, or about the year 2019.

3. a. For France, the growth rate is \( r = 0.051948 \) (about 5.2% per decade) and the general equation is \( P_n = (1.051948)^n 53.9 \).

b. The population predictions are 59.6 million in the year 2000 and 66 million in the year 2020. The error in the year 2000 is 0.34%.

c. In Kenya, the growth rate is \( r = 0.4491 \) (about 44.9% per decade) and the general equation is \( P_n = (1.4491)^n 16.7 \). The population predictions are 35.1 million in the year 2000 and 73.6 million in the year 2020. It takes 1.87 decades, or about 18.7 years, for the population of Kenya to double.

d. The populations become equal in 3.66 decades, so the population of Kenya will first exceed that of France in 2017.

e. It follows that the annual growth rate in France is \( r = 0.00508 \), while in Kenya it is \( r = 0.0378 \).
4. a. \( P_1 = 2150 \) and \( P_2 = 2299 \).

b. The growth rate is zero when \( P = 0 \) and \( P = 4000 \). The maximum growth rate occurs at \( P = 2000 \). This maximum growth is \( g(2000) = 150 \text{ individuals} \times 1000/\text{cm}^3/\text{hr} \). The sketch of \( g(P) \) is below.

c. The two equilibria are \( P = 0 \) and \( P = 4000 \).

\begin{center}
\includegraphics[width=0.5\textwidth]{g_P.png}
\end{center}

5. a. The populations in the first two years are \( P_1 = 110 \) and \( P_2 = 121 \). It takes about 7.3 years for the population to double.

b. For the discrete logistic growth model, \( P_1 = 105 \) and \( P_2 = 109.9875 \).

c. The equilibria are \( P_e = 0 \) or \( P_e = 200 \).
6. a. For the discrete logistic growth model, $p_1 = 130$ and $p_2 = 167.7$.

b. The $p_n$-intercepts are $p_n = 0$ and 4000. The vertex occurs at (2000,1333.3). The points of intersection are (0,0) and (1000,1000). The graph of the updating function and the identity map are shown below.

c. The equilibria are $p_e = 0$ and 1000. The equilibrium at $p_e = 0$ is unstable, and the one at $p_e = 1000$ is stable.

![Graph of the updating function and the identity map]

7. a. The 3 populations are $p_1 = 700$, $p_2 = 860$, and $p_3 = 988$.

b. The equilibrium is $p_e = 1500$. The equilibrium is stable.

8. a. The breathing fraction is $q = 0.120536$, and the functional reserve capacity is $V_r = 2188.9$ ml.

b. The concentration of Helium in the next two breaths are $c_2 = 39.85$ and $c_3 = 35.67$. The equilibrium concentration is $c_e = \gamma = 5.2$ ppm of He, which is a stable equilibrium.
9. a. For the Malthusian growth model with dispersion, \( P_{n+1} = (1 + r)P_n - \mu \), \( r = 0.5 \) and \( \mu = 120 \). The populations in the next two weeks are \( P_3 = 1117.5 \) and \( P_4 = 1556.25 \).

b. The equilibrium is \( P_e = 240 \), and it is unstable.

c. The graph of the updating function and identity map, \( P_{n+1} = P_n \), are shown below. The only point of intersection occurs at the equilibrium found above.
10. a. From the breathing model, \( c_{n+1} = (1 - q)c_n + q\gamma \) and the data \( c_0 = 400, c_1 = 352, \) and \( c_2 = 310, \) we find the constants \( q \) and \( \gamma \) by substitution and the simultaneous solution of two equations and two unknowns. We have

\[
352 = 400(1 - q) + q\gamma \quad \text{and} \quad 310 = 352(1 - q) + q\gamma.
\]

Subtracting the second equation from the first gives 42 = 48(1 - q) or \( 1 - q = \frac{42}{48} = \frac{7}{8}. \) Thus, \( q = \frac{1}{8}. \) This value is substituted into the first equation above to give 352 = 400\( \frac{7}{8} \) + \( \frac{1}{8} \gamma, \) which gives \( \gamma = 16. \)

Thus, the model becomes \( c_{n+1} = \frac{7}{8}c_n + 2, \) and the next 2 breaths satisfy

\[
\begin{align*}
c_3 &= \frac{7}{8} \cdot 310 + 2 = 273.25 \\
c_4 &= \frac{7}{8} \cdot 273.25 + 2 = 241.1
\end{align*}
\]

b. At the equilibria, \( c_e = \frac{7}{8}c_e + 2, \) so \( \frac{1}{8}c_e = 2 \) or \( c_e = 16, \) which is the value of \( \gamma \) as expected. This equilibrium is stable.

c. The graph of the updating function and identity map, \( c_{n+1} = c_n, \) are shown below. The only point of intersection occurs at the equilibrium, \( \gamma \) found above.
11. a. Hassell’s model, $p_{n+1} = H(p_n) = \frac{6p_n}{1+0.001p_n}$, with $p_0 = 2000$ gives $p_1 = 4000$ and $p_2 = 4800$.

b. The equilibria for this model are $p_e = 0$ and 5000.

c. The only intercept is $p = 0$. The horizontal asymptote is $H = 6000$. The graphs of $H(p)$ for $p > 0$ and the identity map, $p_{n+1} = p_n$ are shown below. These functions intersect at the equilibrium $(5000,5000)$.
12. a. The general solution is \( a_n = 1.8^n a_0 = (50000)1.8^n \) with, \( a_5 = (50,000)1.8^5 = 944,784 \).

b. From the selection model, \( p_2 = 0.5806 \) and \( p_0 = 0.419355 \).

c. The equilibria are \( p_e = 0 \) and 1. The limiting fraction for large \( n \) will be \( p_e = 1 \).

d. Below is the graph of the updating function for the fraction of bacteria, \( p_n \), of type \( a \) and the identity map for \( 0 \leq p \leq 1 \).
13. a. The frog stays in the air for 0.4 sec. The frog’s maximum height is 19.6 cm. The graph of the height as a function of time is shown below.

   b. The average velocity of the frog for \( t \in [0.1, 0.2] \) is \( v_1 = 49 \text{ cm/sec} \). The average velocity of the frog for \( t \in [0.1, 0.11] \) is \( v_2 = 93.1 \text{ cm/sec} \).

![Graph of height as a function of time](image)

14. a. The vertical velocity is \( v_0 = 420\sqrt{2} \approx 593.97 \text{ cm/sec} \). The impala is in the air for \( t = \frac{6\sqrt{2}}{7} \approx 1.21218 \text{ sec} \).

   b. The average velocity for the impala between \( t = 0 \) and \( t = 0.5 \) is \( v_{\text{ave}} = 420\sqrt{2} - 245 \approx 348.97 \text{ cm/sec} \).
15. a. The serval can catch any bird flying at heights from 16 to 25 ft or up to 9 ft above the serval.

b. The average velocity of the serval for \( t \in [0, \frac{1}{4}] \) is \( v_{\text{ave}} = 20 \text{ ft/sec} \). The average velocity of the serval for \( t \in [\frac{1}{2}, 1] \) is \( v_{\text{ave}} = 0 \text{ ft/sec} \). The average velocity of the serval for \( t \in [1, \frac{5}{4}] \) is \( v_{\text{ave}} = -12 \text{ ft/sec} \).

c. The instantaneous velocity at \( t = 1 \) satisfies \( v(1) = -8 - 16\Delta t \). As \( \Delta t \to 0 \), \( v(1) = -8 \text{ ft/sec} \).

d. The serval hits the ground at \( t = 2 \). A graph of the height of the serval is below.

![Graph of the serval's height](image)

16. a. The slope of the secant line for \( f(x) = 2x - x^2 \) is \( m_s = -2 - \Delta x \) through the points (2, 0) and (2 + \( \Delta x \), \( f(2 + \Delta x) \)).

b. The slope of the tangent line is \( m_t = -2 \). Thus, the value of the derivative of \( f(x) \) at \( x = 2 \) is \( -2 \). The equation of the tangent line is 

\[
y = -2x + 4.
\]

17. a. There is a vertical asymptote at \( x = 3 \) and a horizontal asymptote at \( y = 0 \). The \( y \)-intercept is \( (0, \frac{3}{2}) \). The graph of the function is shown below.

b. The slope of the secant line for \( f(x) = \frac{2}{3 - x} \) is 

\[
m_s = \frac{2}{1 - \Delta x}.
\]

c. The slope of the tangent line is \( m_t = 2 \). Thus, the value of the derivative of \( f(x) \) at \( x = 2 \) is
2. The equation of the tangent line is
\[ y = 2x - 2. \]

18. a. The \( x \)-intercept is \( (\frac{9}{5}, 0) \), and the \( y \)-intercept is \((0, 3)\). The graph of the function is shown below.

b. The slope of the secant line for \( f(x) = \sqrt{9 - 5x} \) is
\[ m_s = -\frac{5}{\sqrt{4 - 5\Delta x} + 2}. \]

c. The slope of the tangent line is \( m_t = -\frac{5}{2} \). Thus, the value of the derivative of \( f(x) \) at \( x = 1 \) is \(-\frac{5}{2}\). The equation of the tangent line is
\[ y = -\frac{5}{2}x + \frac{9}{2}. \]