1. a. \( f'(x) = -2x^{-2} + (12x^2e^{4x} + 6xe^{4x}) + 6x + \frac{10}{x} \)

b. \( g'(t) = -(-15te^{-3t} + 5e^{-3t}) + (8e^{4t}(t^2 + 5t + 1) + (2t + 5)(2e^{4t} - 7)) + 12e^{-2t} \)

2. a. The linear model is

\[ V = 4d - 25.8 \]

with \( m = 4 \) and \( b = 25.8 \). This model predicts that a tree with a diameter of 15 inches produces \( V(15) = 34.2 \) board feet, while one producing 30.7 board feet is predicted to be 14.13 inches. The graph is below.

b. The allometric model is

\[ V = 0.1583d^{1.979} \]

with \( k = 0.1583 \) and \( a = 1.979 \). This model predicts that a tree with a diameter of 15 inches produces \( V(15) = 33.65 \) board feet, while one producing 30.7 board feet is predicted to be 14.32 inches. The graph is below.

c. Both models are close, but the allometric is better since it passes through the origin. The linear model is negative at \( d = 0 \). The power is approximately 2 because the cross-sectional area (related to \( d^2 \)) supports the weight of the tree reflected in its volume.
3. a. The function \( y = 6x^2 - x^3 \) has the \( x \)-intercepts \((0, 0)\) and \((6, 0)\), the \( y \)-intercept \((0, 0)\), and no asymptotes. The derivative \( y' = 12x - 3x^2 \), which gives a minimum at \((0, 0)\) and a maximum of \((4, 32)\). The graph is below to the left.

\[ \begin{align*}
\text{Graph:} \quad &y = 6x^2 - x^3 \\
\text{Intercepts:} \quad &x \text{-intercepts: } (0, 0), (6, 0), \quad y \text{-intercept: } (0, 0) \\
\text{Asymptotes:} \quad &\text{no asymptotes} \\
\text{Derivative:} \quad &y' = 12x - 3x^2
\end{align*} \]

b. The function \( y = -2x - \frac{8}{x} \) has no \( x \) or \( y \)-intercept. There is a vertical asymptote \( x = 0 \). The derivative \( y' = -2 + \frac{8}{x^2} \), which gives a minimum at \((-2, 8)\) and a maximum at \((2, -8)\). The graph is above to the right.

c. The function \( y = (x - 3)e^x \) has the \( x \)-intercept \((3, 0)\), the \( y \)-intercept \((0, -3)\), and a horizontal asymptote \( y = 0 \) as \( x \to -\infty \). The derivative \( y' = (x - 2)e^x \), which gives a minimum \((2, -e^2) \approx (2, -7.389)\). The graph is below to the left.
4. The derivative is \( r'(t) = -6.4e^{-0.04t} + 9.6e^{-0.06t} \). The initial secretion rate is \( r(0) = 0 \), while the secretion rate at \( t = 40 \) is \( r(40) = 17.79 \text{ mg/100ml of plasma} \). The maximum secretion rate occurs at \( t = 20.27 \text{ yr} \) with a maximum of \( r(20.27) = 23.70 \text{ mg/100ml of plasma} \). There is a horizontal asymptote of \( r = 0 \). The graph is below.
5. a. With \( h(t) = 10(e^{-0.02t} - e^{-0.5t}) \), it follows that \( h(0) = 0 \). Since the \( \lim_{t \to \infty} h(t) = 0 \), there is a horizontal asymptote of \( h = 0 \). The derivative is given by

\[
h(t) = 10(-0.02e^{-0.02t} + 0.5e^{-0.5t}).
\]

The maximum concentration of the hormone occurs at \( t_{\text{max}} = 6.706 \) days with a concentration of \( h(t_{\text{max}}) = 8.395 \). The graph of \( h(t) \) is below.

b. The model predicts \( h(1) = 3.737 \), \( h(10) = 8.120 \), and \( h(30) = 5.488 \). It follows that the percent error at \( t = 30 \) is

\[
\%\text{error} = 100 \left( \frac{5.7 - 5.488}{5.488} \right) = 3.9\%.
\]

The sum of the square errors between the model and the data is

\[
\sum_{i=1}^{3} e_i^2 = (3.737 - 3.5)^2 + (8.120 - 8.2)^2 + (5.488 - 5.7)^2
\]

\[
= 0.056 + 0.0064 + 0.045 = 0.107
\]

6. a. The velocity is given by \( v(t) = h'(t) = v_0 - 980t \).

b. The velocity is zero at \( t = \frac{v_0}{980} \). For the impala to clear the 180 cm fence, the initial velocity must be at least \( v_0 = 593.97 \) cm/sec.

c. The impala stays in the air for 1.21 sec.
7. a. The rate of change of temperature is $\frac{dT}{dt} = -1.35 + 0.54t - 0.03t^2$. At 3 AM, $T'(3) = 0$.

b. The minimum temperature occurs at $t = 3$ (3 AM) and is $T(3) = 14.11$ °C. The maximum temperature occurs at $t = 15$ (3 PM) and is $T(15) = 22.75$ °C.

c. The initial temperature at $t = 0$ (midnight) is $T(0) = 16$ °C, while at $t = 20$ (8 PM) the temperature is $T(20) = 17$ °C. Below is a graph of the temperature.
8. a. The growth of the bacteria satisfies $P_n = 4000(1.025)^n$.
This culture doubles in 28.07 min.

   b. The other culture of bacteria satisfies $B_n = 1000(1.0281)^n$.
These cultures are equal in 457 min.

9. a. The value of $r$ is $r = 0.4676$. The general solution is $P_n = 1.3(1.4676)^n$.

   b. In 2000, the model predicts $P_5 = 8.85$ crabs/m$^2$, which gives an error of 24.7% to high an estimate.

   c. The logistic growth model predicts population densities for the mitten crabs of $P_2 = 1.926$ crabs/m$^2$ in 1996 and $P_3 = 2.799$ crabs/m$^2$ in 1997.

   d. The equilibria are $P_e = 0$ and 12 crabs/m$^2$. The derivative of the updating function is $F'(P) = 1.54 - 0.09P$. For the higher equilibrium, $F'(12) = 0.46$, so this equilibrium is stable with population densities monotonically approaching this value.
10. a. The value of \( q = 0.16 \), which gives the equation,

\[ c_{n+1} = 0.84c_n + 0.8. \]

It follows that \( c_2 = 72.03 \) ppm of He.

b. The equilibrium is \( c_e = 5 \) ppm of He. The derivative of the updating function is \( B'(c) = 0.84 < 1 \), so the equilibrium is stable.

c. The graph of the updating function with identity map is below. The \( B \)-intercept is \( (0, 0.8) \) and the point of intersection is \( (5, 5) \).
11. a. The data give \( r = 0.8 \) and \( \mu = 700 \), so the model is \( P_{n+1} = 0.8P_n + 700 \). It follows that the populations are \( P_3 = 4780 \) in 1993 and \( P_4 = 4524 \) in 1994.

b. The only equilibrium is \( P_e = 3500 \). This equilibrium is stable (slope of updating function is 0.8), so the model predicts that the population of moths will stabilize at 3500 moths on the island.

c. Below is a graph of the updating function and identity map.

![Updating Function for Moths](image)

12. a. The Malthusian growth model for the U. S. population is

\[
P_n = 76.0(1.1793)^n.
\]

b. In 1960, \( P_6 = 204.4 \) million. The error is

\[
%error = \left( \frac{204.4 - 179.3}{179.3} \right) 100 = 14.0\%.
\]

c. From the logistic growth model, in 1910, \( P_1 = 89.9 \) million and in 1930, \( P_3 = 123.5 \) million.
d. The equilibria for the logistic growth model are \( P_e = 0 \) and 449.0 million.

\[
F'(P) = 1.22 - 0.00098P,
\]

so \( F'(449) = 0.78 < 1 \), which implies that \( P_e = 449.0 \) is a stable equilibrium with solutions monotonically approaching this equilibrium.

13. a. The populations are \( P_1 = 800e^{-0.4} \approx 536.26 \) and \( P_2 = 502.2 \).

b. The derivative of \( R(P) \) is \( R'(P) = 8(1 - .004P)e^{-0.004P} \). The maximum of \( R(P) \) occurs at \( P = 250 \) with \( R(250) = 2000e^{-1} = 735.76 \). As \( P \to \infty \), the exponential dominates the polynomial part, so \( R(P) \to 0 \). The graph of the Ricker’s function is below.

c. The equilibria are \( P_e = 0 \) and \( P_e = 250\ln(8) = 519.86 \). At \( P_e = 0 \), \( R'(0) = 8 > 1 \), so this equilibrium is unstable with solutions monotonically growing away from \( P_e = 0 \). At \( P_e = 519.86 \), \( R'(519.86) = -1.079 < -1 \), so this equilibrium is unstable with solutions oscillating and moving away from \( P_e = 519.86 \).

![Ricker's Updating Function](image)

14. a. With \( h = 0.5 \), \( P_1 = 402.4 \) and \( P_2 = 1144.3 \).
b. The equilibria with $h = 0.5$ are $P_e = 0$ and $1000\ln\left(\frac{10}{3}\right) \simeq 1204.0$. The derivative of the updating function is

$$R'(P) = 5(1 - 0.001P)e^{-0.001P} - 0.5.$$ 

At $P_e = 0$, $R'(0) = 4.5 > 1$, so this equilibrium is unstable, monotonically growing away from 0. At $P_e = 1204$, $R'(1204) = -0.806$, so this equilibrium is stable, oscillating toward the equilibrium.

c. Solving $P_e = 5Pe^{-0.001Pe} - hPe$ gives either $P_e = 0$ or

$$P_e = 1000\ln\left(\frac{5}{1 + h}\right),$$

which is zero when $h = 4$. Thus, a fishing intensity of $h \geq 4$ leads to extinction.

15. a. The populations are $P_1 = 11,790$ and $P_2 = 13,706$.

b. The derivative of $G(P)$ is $G'(P) = 2 - 0.1\ln(P)$. The maximum occurs at $P_{max} = e^{20} = 4.85 \times 10^8$ with $G(P_{max}) = 0.1e^{20} = 4.85 \times 10^7$. The $P$-intercept is $P = e^{21} = 1.32 \times 10^9$. The graph is shown below.

c. The equilibrium is $P_e = e^{11} = 59,874$. At $P_e = e^{11}$, $G'(e^{11}) = 0.9 < 1$, so the equilibrium is stable with solutions monotonically approaching $P_e = e^{11}$.