1. a. \( f'(x) = 8x^3 - 4e^{-2x} - 5(x^2 - \ln(x))^4\left(2x - \frac{1}{x}\right) - \frac{(x + e^{-x})12x^2 - 4x^3(1 - e^{-x})}{(x + e^{-x})^2} \)

b. \( g'(x) = -3x^2e^{-3x} + 2xe^{-3x} + \frac{2}{x} + 15(x + 1)^{-4} - 8xe^{-x^2} \)

2. a. The linear model is \( V = 4d - 25.8 \) with \( m = 4 \) and \( b = 25.8 \). This model predicts that a tree with a diameter of 15 inches produces \( V(15) = 34.2 \) board feet, while one producing 30.7 board feet is predicted to be 14.13 inches. The graph is below.

b. The allometric model is \( V = 0.1583d^{1.979} \) with \( k = 0.1583 \) and \( a = 1.979 \). This model predicts that a tree with a diameter of 15 inches produces \( V(15) = 33.65 \) board feet, while one producing 30.7 board feet is predicted to be 14.32 inches. The graph is below.

c. Both models are close, but the allometric is better since it passes through the origin. The linear model is negative at \( d = 0 \). The power is approximately 2 because the cross-sectional area (related to \( d^2 \)) supports the weight of the tree reflected in its volume.
3. a. The function \( y = (x - 3)e^x \) has the \( x \)-intercept \((3, 0)\), the \( y \)-intercept \((0, -3)\), and a horizontal asymptote \( y = 0 \) as \( x \to -\infty \). The derivative \( y' = (x - 2)e^x \), which gives a minimum \((2, -e^2) \simeq (2, -7.389)\). The graph is below to the left.

\[ \begin{align*} 
&y = (x - 3)e^x \\
&y' = (x - 2)e^x \\
&y = -2x - \frac{8}{x} \\
&y' = -2 + \frac{8}{x^2} \\ 
\end{align*} \]

b. The function \( y = -2x - \frac{8}{x} \) has no \( x \) or \( y \)-intercept. There is a vertical asymptote \( x = 0 \). The derivative \( y' = -2 + \frac{8}{x^2} \), which gives a minimum at \((-2, 8)\) and a maximum at \((2, -8)\). The graph is above to the right.
4. a. With \( h(t) = 10(e^{-0.02t} - e^{-0.5t}) \), it follows that \( h(0) = 0 \). Since the \( \lim_{t \to \infty} h(t) = 0 \), there is a horizontal asymptote of \( h = 0 \). The derivative is given by

\[ h(t) = 10(-0.02e^{-0.02t} + 0.5e^{-0.5t}). \]

The maximum concentration of the hormone occurs at \( t_{\text{max}} = 6.706 \) days with a concentration of \( h(t_{\text{max}}) = 8.395 \). The graph of \( h(t) \) is below.

b. The model predicts \( h(1) = 3.737 \), \( h(10) = 8.120 \), and \( h(30) = 5.488 \). It follows that the percent error at \( t = 30 \) is

\[ \%\text{error} = 100 \left( \frac{5.488 - 5.7}{5.488} \right) = 3.9\%. \]

The sum of the square errors between the model and the data is

\[
\sum_{i=1}^{3} e_i^2 = (3.737 - 3.5)^2 + (8.120 - 8.2)^2 + (5.488 - 5.7)^2 \\
= 0.056 + 0.0064 + 0.045 = 0.107
\]

5. a. The growth of the bacteria satisfies \( P_n = 4000(1.025)^n \). This culture doubles in 28.07 min.

b. The other culture of bacteria satisfies \( B_n = 1000(1.0281)^n \). These cultures are equal in 457 min.
6. a. The next two populations, $P_1 = 1032$ and $P_2 = 1065$.

   b. The growth rate is zero at $P = 0$ and $P = 5000$. The growth rate is a maximum at $P_v = 2500$ with a growth rate of $g(P_v) = 50$ individuals/unit time. A sketch of the graph is below.

   c. The equilibria for this logistic equation are $P_e = 0$ and 5000 (when the growth rate is zero). The derivative of $F(P)$ is

   $$F'(P) = 1.04 - 0.000016P.$$  

   At $P_e = 0$, $F'(0) = 1.04 > 1$, so this equilibrium is unstable, solutions monotonically growing away from 0. At $P_e = 5000$, $F'(5000) = 0.96 < 1$, so this equilibrium is stable, solutions monotonically approach 5000.
7. a. The velocity is given by \( v(t) = h'(t) = v_0 - 980t \).

b. The velocity is zero at \( t = \frac{v_0}{980} \). For the impala to clear the 180 cm fence, the initial velocity must be at least \( v_0 = 593.97 \) cm/sec.

c. The impala stays in the air for 1.21 sec.

8. a. The value of \( q = 0.16 \), which gives the equation,

\[
C_{n+1} = 0.84C_n + 0.8.
\]

It follows that \( C_2 = 72.03 \) ppm of He.

b. The equilibrium is \( C_e = 5 \) ppm of He. The derivative of the updating function is \( B'(c) = 0.84 < 1 \), so the equilibrium is stable.

c. The graph of the updating function with identity map is below. The \( B\)-intercept is \((0, 0.8)\) and the point of intersection is \((5, 5)\).
9. a. The next two generations are $P_1 = 800$ and $P_2 = 512$.

b. The only intercept is $(0,0)$. There is a horizontal asymptote at $H = 0$, since $\lim_{P \to \infty} H(P) = 0$. The maximum occurs at $(200, 800)$. The graph is below.

c. There are two equilibria. At $P_e = 0$, $H'(0) = 16 > 1$, so this equilibrium is unstable, monotonically growing away from 0. At $P_e = 600$, $H'(600) = -0.5$, so this equilibrium is stable, oscillating toward the equilibrium.
10. a. With $h = 0.5$, $P_1 = 402.4$ and $P_2 = 1144.3$.

b. The equilibria with $h = 0.5$ are $P_e = 0$ and $1000 \ln \left( \frac{10}{3} \right) \simeq 1204.0$. The derivative of the updating function is

$$R'(P) = 5(1 - 0.001P)e^{-0.001P} - 0.5.$$ 

At $P_e = 0$, $R'(0) = 4.5 > 1$, so this equilibrium is unstable, monotonically growing away from $0$. At $P_e = 1204$, $R'(1204) = -0.806$, so this equilibrium is stable, oscillating toward the equilibrium.

c. Solving $P_e = 5P_e e^{-0.001P_e} - hP_e$ gives either $P_e = 0$ or

$$P_e = 1000 \ln \left( \frac{5}{1 + h} \right),$$

which is zero when $h = 4$. Thus, a fishing intensity of $h \geq 4$ leads to extinction.

11. a. The Malthusian growth model for the U. S. population is

$$P_n = 76.0(1.1793)^n.$$ 

b. In 1960, $P_6 = 204.4$ million. The error is

$$\% \text{error} = \left( \frac{204.4 - 179.3}{179.3} \right) 100 = 14.0\%.$$ 

c. From the logistic growth model, in 1910, $P_1 = 89.9$ million and in 1930, $P_3 = 123.5$ million.

d. The equilibria for the logistic growth model are $P_e = 0$ and 449.0 million.

$$F'(P) = 1.22 - 0.00098P,$$

so $F'(449) = 0.78 < 1$, which implies that $P_e = 449.0$ is a stable equilibrium with solutions monotonically approaching this equilibrium.