Find the derivatives of the following functions:

1. \( f(x) = \frac{x^3 - \ln(x)}{1 - x^2} + \frac{2}{x^2} \),

2. \( f(x) = \frac{x^2 - e^{-x}}{3x + 1} + xe^{-x} \),

3. \( f(x) = \frac{\sqrt{x}}{2 + x} - \frac{1}{e^{3x}} \),

4. \( f(x) = \frac{x^2 + 5}{x^2 - e^x} - \frac{xe^{2x}}{2x + 1} \).

Find the derivative and sketch the curves of the functions below. Give the domain of each of the functions. List all maxima and minima for each graph. Also, give the \( x \) and \( y \)-intercepts and any asymptotes if they exist.

5. \( y = \frac{x^2}{x + 1} \),

6. \( y = \frac{e^x}{x + 1} \),

7. \( y = \frac{x^2 - 2x + 2}{x - 1} \),

8. \( y = \frac{x^2}{x^2 + 1} \).
9. Consider the chalone model for mitosis given by the equation

\[ P_{n+1} = f(P_n) = \frac{2P_n}{1 + (bP_n)^c}, \]

where \( b = 0.05 \) and \( c = 2 \).

a. Let \( P_0 = 10 \), then find \( P_1, P_2, \) and \( P_3 \).

b. Sketch a graph of \( f(P) \) with the identity function for \( P \geq 0 \), showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

10. Repeat Exercise 10 with \( b = 0.02 \) and \( c = 5 \).
11. Some entomologists use Hassell’s model for studying the population of insects. Let $P_n$ be the population of a species of beetle in week $n$ and suppose that Hassell’s model is given by

$$P_{n+1} = H(P_n) = \frac{aP_n}{1 + bP_n}.$$ 

Suppose that the best fit to a set of data gives $a = 5$ and $b = 0.004$ for this species of beetle.

a. Let $P_0 = 100$, then find $P_1$, $P_2$, and $P_3$.

b. Sketch a graph of $H(P)$ with the identity function for $P \geq 0$, showing the intercepts and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.