1. \( f'(x) = (x^3 - 3x^2 + 7)(4x^3 - 4x + 6) + (x^4 - 2x^2 + 6x - 1)(3x^2 - 6x), \)
2. \( f'(x) = 3(x^2 - e^{2x} + 1) + (3x + 8)(2x - 2e^{2x}), \)
3. \( f'(x) = (2x - x^2)e^{-x} + \frac{21}{2}x^{-1/2}, \)
4. \( f'(x) = x^{-3}(1 - 2 \ln(x)) - 2e^{2x}(x^2 + x - 1). \)
5. \( y' = 3(1 - 0.02x)e^{-0.02x}. \) Domain: All \( x. \) Only intercept: \((0, 0).\) Horizontal asymptote: \( y = 0.\) Maximum at \((50, 150e^{-1}) \simeq (50, 55.2).\) Graph is below to the left.

6. \( y' = (3 - x)e^{-x}. \) Domain: All \( x. \) y-intercept: \((0, -2).\) x-intercept: \((2, 0).\) Horizontal asymptote: \( y = 0.\) Maximum at \((3, e^{-3}) \simeq (3, 0.0498).\) Graph is above to the right.

7. \( y' = x^{-2}(1 - \ln(x)). \) Domain: \( x > 0. \) Only an x-intercept: \((1, 0).\) Vertical asymptote: \( x = 0.\) Maximum at \((e, e^{-1}) \simeq (2.72, 0.368).\) Graph is below to the left.

8. \( y' = (x^2 + 2x - 3)e^x. \) Domain: All \( x. \) y-intercept: \((0, -3).\) x-intercepts: \((\pm \sqrt{3}, 0).\) Horizontal asymptote: \( y = 0.\) Maximum at \((-3, 6e^{-3}) \simeq (-3, 0.2987)\) and minimum at \((1, -2e) \simeq (1, -5.437).\) Graph is above to the right.
9. a. $P_1 = 335$, $P_2 = 439$, and $P_3 = 379$.

b. Below is a sketch of $R(P)$ with the identity function. Only intercept: $(0, 0)$. Maximum: $(250, 1250e^{-1}) \simeq (250, 459.8)$. Horizontal asymptote: $R = 0$.

c. The equilibria are $P_e = 0$ and $250 \ln(5) \simeq 402.36$. At $P_e = 0$, $R'(0) = 5 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 250 \ln(5)$, $R'(250 \ln(5)) = 1 - \ln(5) \simeq -0.61$, which means the solution oscillates, but approaches the equilibrium, so is stable.

10. a. $P_1 = 655$, $P_2 = 1414$, and $P_3 = 669$.

b. Below is a sketch of $R(P)$ with the identity function. Only intercept: $(0, 0)$. Maximum: $(500, 4000e^{-1}) \simeq (500, 1471.5)$. Horizontal asymptote: $R = 0$.

c. The equilibria are $P_e = 0$ and $500 \ln(8) \simeq 1039.7$. At $P_e = 0$, $R'(0) = 8 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 500 \ln(8)$, $R'(500 \ln(8)) = 1 - \ln(8) \simeq -1.08$, which means the solution oscillates and grows away from the equilibrium, so is unstable.
11. a. $P_1 = 277$, $P_2 = 498$, and $P_3 = 486$.

b. The equilibria are $P_e = 0$ and $500 \ln(8/3) \simeq 490.4$. At $P_e = 0$, $F'(0) = 3.5 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 500 \ln(8/3)$, $F'(500 \ln(8/3)) = 1 - 1.5 \ln(8/3) \simeq -0.47$, which means the solution oscillates, but approaches the equilibrium, so is stable.

c. For $h = 1$, the equilibria are $P_e = 0$ and $500 \ln(2) \simeq 346.6$. At $P_e = 0$, $F'(0) = 3 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 500 \ln(2)$, $F'(500 \ln(2)) = 1 - 2 \ln(2) \simeq -0.386$, which means the solution oscillates, but approaches the equilibrium, so is stable.

For $h = 2$, the equilibria are $P_e = 0$ and $500 \ln(4/3) \simeq 143.8$. At $P_e = 0$, $F'(0) = 2 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 500 \ln(4/3)$, $F'(500 \ln(4/3)) = 1 - 3 \ln(4/3) \simeq 0.137$, which means the solution monotonically approaches the equilibrium, so is stable.

d. The fish will go extinct for any $h \geq 3$.

12. Maximum air velocity at $r = 2R/3$ with a maximum air velocity of $v(2R/3) = 4AR^3/27$. 