This Lecture Activity has you actively work with the lecture notes presented in class and available on my website. This activity is due by **Mon. Sep 27 by noon**. The problems below require written answers, which are entered into **Gradescope**.

Note: For full credit you must show intermediate steps in your calculations.

For all of the problems below consider the matrix:

$$A = \begin{pmatrix} 2 & -6 & 4 \\ -2 & 1 & -2 \\ 1 & 6 & -1 \end{pmatrix}.$$

1. (4pts) Find the $\|A\|_1$, $\|A\|_2$, and $\|A\|_\infty$. (Slide Fundamental 16)

2. (4pts) Find a matrix P, such that $P^{-1}AP = D$, where D has the form:

$$D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix},$$

and $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$. (Slides Fundamental 21–22)

3. (4pts) With D the diagonal matrix defined above, find the solution $\mathbf{y}(t)$ to the IVP given by:

$$\dot{\mathbf{y}} = D\mathbf{y}, \qquad \text{with} \quad \mathbf{y}(0) = [2, -1, 1]^T.$$

Describe the type of nodes for the origin in the y_1 vs y_2, y_1 vs y_3, and y_2 vs y_3 phase planes and sketch these three 2D phase portraits. (Slides Fundamental 22-26)

4. (4pts) Returning to the original matrix A, find the general solution to the IVP:

$$\dot{\mathbf{x}} = A\mathbf{x}, \qquad \text{with} \quad \mathbf{x}(0) = \mathbf{x}_0.$$

(Slide Fundamental 25)