```
> with(LinearAlgebra) :
> A := Matrix([[-7,-5,-3], [2,-2,-3], [0, 1, 0]]);
```

$$A := \begin{bmatrix} -7 & -5 & -3 \\ 2 & -2 & -3 \\ 0 & 1 & 0 \end{bmatrix} \quad (1)$$

```
> J := JordanForm(A);
```

$$J := \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix} \quad (2)$$

```
> Q := JordanForm(A, output='Q');
```

$$Q := \begin{bmatrix} 6 & -4 & 1 \\ -6 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} \quad (3)$$

> $Q^{-1} \cdot A \cdot Q;$

$$\begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix} \quad (4)$$

> $Q^{-1};$

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{3}{2} \\ 1 & 2 & 3 \end{bmatrix} \quad (5)$$

```
> Eigenvectors(A);
```

$$\begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 & 0 & 0 \\ -3 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (6)$$

```
>
```

Diagonalization for Complex example

```
> A2 := Matrix([[0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1], [-4,-8,-8,-4]]);
```

$$A2 := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -8 & -8 & -4 \end{bmatrix} \quad (7)$$

```
> Eigenvectors(A2); eigenvectors(A2);
```

$$\left[\begin{array}{c} -1+I \\ -1+I \\ -1-I \\ -1-I \end{array}\right], \left[\begin{array}{cccc} \frac{1}{4}-\frac{I}{4} & 0 & \frac{1}{4}+\frac{I}{4} & 0 \\ \frac{I}{2} & 0 & -\frac{I}{2} & 0 \\ -\frac{1}{2}-\frac{I}{2} & 0 & -\frac{1}{2}+\frac{I}{2} & 0 \\ 1 & 0 & 1 & 0 \end{array}\right]$$

$$eigenvectors\left(\left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -8 & -8 & -4 \end{array}\right]\right) \quad \textbf{(8)}$$

```
> J2 := JordanForm(A2); Q := JordanForm(A2, output='Q');
```

$$J2 := \left[\begin{array}{cccc} -1-I & 1 & 0 & 0 \\ 0 & -1-I & 0 & 0 \\ 0 & 0 & -1+I & 1 \\ 0 & 0 & 0 & -1+I \end{array}\right]$$

$$Q := \left[\begin{array}{cccc} -\frac{1}{2}+\frac{I}{2} & \frac{1}{2}+I & -\frac{1}{2}-\frac{I}{2} & \frac{1}{2}-I \\ 1 & -I & 1 & I \\ -1-I & I & -1+I & -I \\ 2I & -2I & -2I & 2I \end{array}\right] \quad \textbf{(9)}$$

```
>
```

We note that A2 is a companion matrix, so we obtain 2 real eigenvectors from the real and imaginary parts of the eigenvector. Let z be eigenvalue, then eigenvector is [1, z, z^2, z^3]^T

```
> v1 := Vector([1, -1 + I, (-1 + I)^2, (-1 + I)^3]);
```

$$v1 := \left[\begin{array}{c} 1 \\ -1+I \\ -2I \\ 2+2I \end{array}\right] \quad \textbf{(10)}$$

This eigenvalue has algebraic multiplicity of 2 and geometric multiplicity of 1, so must go to the higher null space (A - lambda I)v2 = v1.

```
> with(Student[LinearAlgebra]) :
> A2ev := Matrix([[1 - I, 1, 0, 0], [0, 1 - I, 1, 0], [0, 0, 1 - I, 1], [-4,-8,-8,-3 - I]]);
```

(11)

$$A2ev := \begin{bmatrix} 1-I & 1 & 0 & 0 \\ 0 & 1-I & 1 & 0 \\ 0 & 0 & 1-I & 1 \\ -4 & -8 & -8 & -3-I \end{bmatrix} \quad (11)$$

> *ReducedRowEchelonForm*(⟨*A2ev*|*v1*⟩);

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{4}+\frac{I}{4} & \frac{3}{2}+\frac{3\,I}{2} \\ 0 & 1 & 0 & -\frac{I}{2} & -2 \\ 0 & 0 & 1 & \frac{1}{2}+\frac{I}{2} & 1-I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

Need to find a v2 = [w1,w2,w3,w4]^T that satisfies the RREF above. Take w4 = 2, then we easily see that w1 = 2+i, w2 = -2+i, w3 = -2i.

Create P from the real and imaginary parts of v1 and v2.

> *P* := *Matrix*([[1, 0, 2, 1], [-1, 1, -2, 1], [0, -2, 0, -2], [2, 2, 2, 0]]);

$$P := \begin{bmatrix} 1 & 0 & 2 & 1 \\ -1 & 1 & -2 & 1 \\ 0 & -2 & 0 & -2 \\ 2 & 2 & 2 & 0 \end{bmatrix} \quad (13)$$

> *Pinv* := *MatrixInverse*(*P*);

$$Pinv := \begin{bmatrix} 2 & 3 & \frac{5}{2} & 1 \\ -1 & -1 & -1 & 0 \\ -1 & -2 & -\frac{3}{2} & -\frac{1}{2} \\ 1 & 1 & \frac{1}{2} & 0 \end{bmatrix} \quad (14)$$

> *Pinv* • *A2* • *P*;

$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix} \quad (15)$$

>