>

Example for Jordan Canonical Form using Maple

> *with*(*LinearAlgebra*) :

> $A := Matrix([[0, 1, 0], [0, 0, 1], [2, 3, 0]]);$

$$A := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 3 & 0 \end{bmatrix} \quad (1)$$

> $CharacteristicPolynomial(A, z);$

$$z^3 - 3z - 2 \quad (2)$$

> $Eigenvectors(A);$

$$\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} \frac{1}{4} & 1 & 0 \\ \frac{1}{2} & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad (3)$$

We see that lambda_1 = -1 has algebraic multiplicity of 2 and geometric multiplicity of 1, while lambda_2 = 2 has algebraic and geometric multiplicity of 1.

> $J := JordanForm(A, output='J'); Q := JordanForm(A, output='Q');$

$$J := \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$Q := \begin{bmatrix} \frac{1}{9} & \frac{2}{3} & \frac{8}{9} \\ \frac{2}{9} & -\frac{2}{3} & -\frac{2}{9} \\ \frac{4}{9} & \frac{2}{3} & -\frac{4}{9} \end{bmatrix} \quad (4)$$

The JordanForm command appears to require output variables of J and Q, proving the Jordan Canonical Form, J, and the Transition Matrix, Q.

> $QI := MatrixInverse(Q);$

$$QI := \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & \frac{1}{2} \\ 1 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \quad (5)$$

> $J := QI \cdot A \cdot Q;$

$$J := \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \quad (6)$$

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This example has the companion matrix A. We note that if z is an eigenvalue, then it has an eigenvector,
v = [1, z, z^2, ... , z^n]^T. Thus, we have the following eigenvectors.

For lamba_1 = 2 we get v1 and for lamba_2 = -1 we get v2.

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> v1 := Vector([1, 2, 4]); v2 := Vector([1,-1, 1]);
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$$v1 := \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$v2 := \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad (7)$$

These form two columns of our matrix P. The last column comes from the second null space for lamba_2 = -1, where (A - lambda_2I)v3 = v2. Solving

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> with(Student[LinearAlgebra]) :
> Aev := Matrix([[1, 1, 0], [0, 1, 1], [2, 3, 1]]);
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$$Aev := \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} \quad (8)$$

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> ReducedRowEchelonForm(⟨Aev|v2⟩);
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$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

The higher null space eigenvector solves this above, so we can obtain v3 = [2, -1, 0], thus we get our transformation matrix

```
> P := Matrix([[1, 1, 2], [2,-1,-1], [4, 1, 0]]);
```

$$P := \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & -1 \\ 4 & 1 & 0 \end{bmatrix} \quad (10)$$

```
> Pinv := MatrixInverse(P);
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$$Pinv := \begin{bmatrix} \frac{1}{9} & \frac{2}{9} & \frac{1}{9} \\ -\frac{4}{9} & -\frac{8}{9} & \frac{5}{9} \\ \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \quad \textbf{(11)}$$

> $Pinv \cdot A \cdot P;$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \quad \textbf{(12)}$$

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