> *with*(*LinearAlgebra*) :

> $A := Matrix([[-7,-5,-3],[2,-2,-3],[0,1,0]]);$

$$A := \begin{bmatrix} -7 & -5 & -3 \\ 2 & -2 & -3 \\ 0 & 1 & 0 \end{bmatrix} \quad \textbf{(1)}$$

> $J := JordanForm(A);$

$$J := \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix} \quad \textbf{(2)}$$

> $Q := JordanForm(A, output='Q');$

$$Q := \begin{bmatrix} 6 & -4 & 1 \\ -6 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} \quad \textbf{(3)}$$

> $Q^{-1} \cdot A \cdot Q;$

$$\begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix} \quad \textbf{(4)}$$

> $Q^{-1};$

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{3}{2} \\ 1 & 2 & 3 \end{bmatrix} \quad \textbf{(5)}$$

> $Eigenvectors(A);$

$$\begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 & 0 & 0 \\ -3 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \textbf{(6)}$$

> *with*(*linalg*) :

> $B := matrix([[-7,-5,-3],[2,-2,-3],[0,1,0]]);$

$$B := \begin{bmatrix} -7 & -5 & -3 \\ 2 & -2 & -3 \\ 0 & 1 & 0 \end{bmatrix} \quad \textbf{(7)}$$

> $eigenvectors(B);$

$$\left[-3, 3, \left\{\begin{bmatrix} 3 & -3 & 1 \end{bmatrix}\right\}\right] \quad \textbf{(8)}$$

Diagonalization for Complex example

> $A2 := Matrix([[0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1], [-4,-8,-8,-4]]);$

$$A2 := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -8 & -8 & -4 \end{bmatrix} \quad \textbf{(9)}$$

> $Eigenvectors(A2); eigenvectors(A2);$

$$\begin{bmatrix} -1+I \\ -1+I \\ -1-I \\ -1-I \end{bmatrix}, \begin{bmatrix} \frac{1}{4} - \frac{I}{4} & 0 & \frac{1}{4} + \frac{I}{4} & 0 \\ \frac{I}{2} & 0 & -\frac{I}{2} & 0 \\ -\frac{1}{2} - \frac{I}{2} & 0 & -\frac{1}{2} + \frac{I}{2} & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\left[-1+I, 2, \left\{\left[\begin{matrix} 1 & -1+I & -2I & 2+2I \end{matrix}\right]\right\}\right], \left[-1-I, 2, \left\{\left[\begin{matrix} 1 & -1-I & 2I & 2-2I \end{matrix}\right]\right\}\right] \quad \textbf{(10)}$$

> $J2 := JordanForm(A2); Q := JordanForm(A2, output='Q');$

$$J2 := \begin{bmatrix} -1-I & 1 & 0 & 0 \\ 0 & -1-I & 0 & 0 \\ 0 & 0 & -1+I & 1 \\ 0 & 0 & 0 & -1+I \end{bmatrix}$$

$$Q := \begin{bmatrix} -\frac{1}{2} + \frac{I}{2} & \frac{1}{2} + I & -\frac{1}{2} - \frac{I}{2} & \frac{1}{2} - I \\ 1 & -I & 1 & I \\ -1-I & I & -1+I & -I \\ 2I & -2I & -2I & 2I \end{bmatrix} \quad \textbf{(11)}$$

> $B2 := Matrix([[-1, 1, 1, 0], [-1,-1, 0, 1], [0, 0,-1, 1], [0, 0,-1,-1]]);$

$$B2 := \begin{bmatrix} -1 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix} \quad \textbf{(12)}$$

> $P := Matrix([[p11, p12, p13, p14], [p21, p22, p23, p24], [p31, p32, p33, p34], [p41, p42, p43, p44]]);$

$$P := \begin{bmatrix} p11 & p12 & p13 & p14 \\ p21 & p22 & p23 & p24 \\ p31 & p32 & p33 & p34 \\ p41 & p42 & p43 & p44 \end{bmatrix} \quad \textbf{(13)}$$

> $C2 := A2 \cdot P; D2 := P \cdot B2; Pe := C2 - D2;$

$C2 := [[p21, p22, p23, p24],$
$[p31, p32, p33, p34],$
$[p41, p42, p43, p44],$
$[-4\,p11 - 8\,p21 - 8\,p31 - 4\,p41, -4\,p12 - 8\,p22 - 8\,p32 - 4\,p42, -4\,p13 - 8\,p23 - 8\,p33 - 4\,p43, -4\,p14 - 8\,p24 - 8\,p34 - 4\,p44]]$

$$D2 := \begin{bmatrix} -p11 - p12 & p11 - p12 & p11 - p13 - p14 & p12 + p13 - p14 \\ -p21 - p22 & p21 - p22 & p21 - p23 - p24 & p22 + p23 - p24 \\ -p31 - p32 & p31 - p32 & p31 - p33 - p34 & p32 + p33 - p34 \\ -p41 - p42 & p41 - p42 & p41 - p43 - p44 & p42 + p43 - p44 \end{bmatrix}$$

$Pe := [[p21 + p11 + p12, p22 - p11 + p12, p23 - p11 + p13 + p14, p24 - p12 - p13 + p14],$
$[p31 + p21 + p22, p32 - p21 + p22, p33 - p21 + p23 + p24, p34 - p22 - p23 + p24],$
$[p41 + p31 + p32, p42 - p31 + p32, p43 - p31 + p33 + p34, p44 - p32 - p33 + p34],$
$[-4\,p11 - 8\,p21 - 8\,p31 - 3\,p41 + p42, -4\,p12 - 8\,p22 - 8\,p32 - 3\,p42 - p41, -4\,p13 - 8\,p23 - 8\,p33 - 3\,p43 - p41 + p44, -4\,p14 - 8\,p24 - 8\,p34 - 3\,p44 - p42 - p43]]$ **(14)**

> $p11 :='p11';$

$$p11 := p11 \quad \textbf{(15)}$$

> $eq1 := Pe[1, 1] = 0; eq2 := Pe[1, 2] = 0; eq3 := Pe[1, 3] = 0; eq4 := Pe[1, 4] = 0; eq5 := Pe[2, 1] = 0; eq6 := Pe[2, 2] = 0; eq7 := Pe[2, 3] = 0; eq8 := Pe[2, 4] = 0; eq9 := Pe[3, 1] = 0; eq10 := Pe[3, 2] = 0; eq11 := Pe[3, 3] = 0; eq12 := Pe[3, 4] = 0; eq13 := Pe[4, 1] = 0; eq14 := Pe[4, 2] = 0; eq15 := Pe[4, 3] = 0; eq16 := Pe[4, 4] = 0;$

$$eq1 := p21 + p11 + p12 = 0$$
$$eq2 := p22 - p11 + p12 = 0$$
$$eq3 := p23 - p11 + p13 + p14 = 0$$
$$eq4 := p24 - p12 - p13 + p14 = 0$$
$$eq5 := p31 + p21 + p22 = 0$$
$$eq6 := p32 - p21 + p22 = 0$$
$$eq7 := p33 - p21 + p23 + p24 = 0$$
$$eq8 := p34 - p22 - p23 + p24 = 0$$
$$eq9 := p41 + p31 + p32 = 0$$
$$eq10 := p42 - p31 + p32 = 0$$
$$eq11 := p43 - p31 + p33 + p34 = 0$$
$$eq12 := p44 - p32 - p33 + p34 = 0$$
$$eq13 := -4\,p11 - 8\,p21 - 8\,p31 - 3\,p41 + p42 = 0$$
$$eq14 := -4\,p12 - 8\,p22 - 8\,p32 - 3\,p42 - p41 = 0$$
$$eq15 := -4\,p13 - 8\,p23 - 8\,p33 - 3\,p43 - p41 + p44 = 0$$
$$eq16 := -4\,p14 - 8\,p24 - 8\,p34 - 3\,p44 - p42 - p43 = 0 \quad \textbf{(16)}$$

> $solve(\{eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10, eq11, eq12, eq13, eq14, eq15, eq16\}, \{p11, p12, p13, p14, p21, p22, p23, p24, p13, p32, p33, p34, p41, p42, p43, p44\});$

$$\left\{p11 = p24 + p23 + 2\,p14 - \frac{p31}{2}, p12 = \frac{p31}{2}, p13 = p24 + p14 - \frac{p31}{2}, p14 = p14, p21 = -p24 - p23 - 2\,p14, p22 = p23 + 2\,p14 - p31 + p24, p23 = p23, p24 = p24, p32 = -2\,p23 - 4\,p14 + p31 - 2\,p24, p33 = -2\,p23 - 2\,p14 - 2\,p24, p34 = 2\,p23 + 2\,p14 - p31, p41 = 2\,p23 + 4\,p14 - 2\,p31 + 2\,p24, p42 = 2\,p23 + 4\,p14 + 2\,p24, p43 = 2\,p31 + 2\,p24, p44 = -6\,p23 - 8\,p14 + 2\,p31 - 4\,p24\right\} \quad \textbf{(17)}$$

> P2 := Matrix([[1, 1,-1, 0], [-2, 0, 2, 0], [2,-2,-4, 2], [0, 4, 4,-8]]);

$$P2 := \begin{bmatrix} 1 & 1 & -1 & 0 \\ -2 & 0 & 2 & 0 \\ 2 & -2 & -4 & 2 \\ 0 & 4 & 4 & -8 \end{bmatrix} \quad \textbf{(18)}$$

> PI2 := MatrixInverse(P2);

$$\Pi 2 := \begin{bmatrix} -1 & -2 & -1 & -\frac{1}{4} \\ 1 & \frac{1}{2} & 0 & 0 \\ -1 & -\frac{3}{2} & -1 & -\frac{1}{4} \\ 0 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} \end{bmatrix} \quad \textbf{(19)}$$

> PI2 · A2 · P2;

$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix} \quad \textbf{(20)}$$

>