

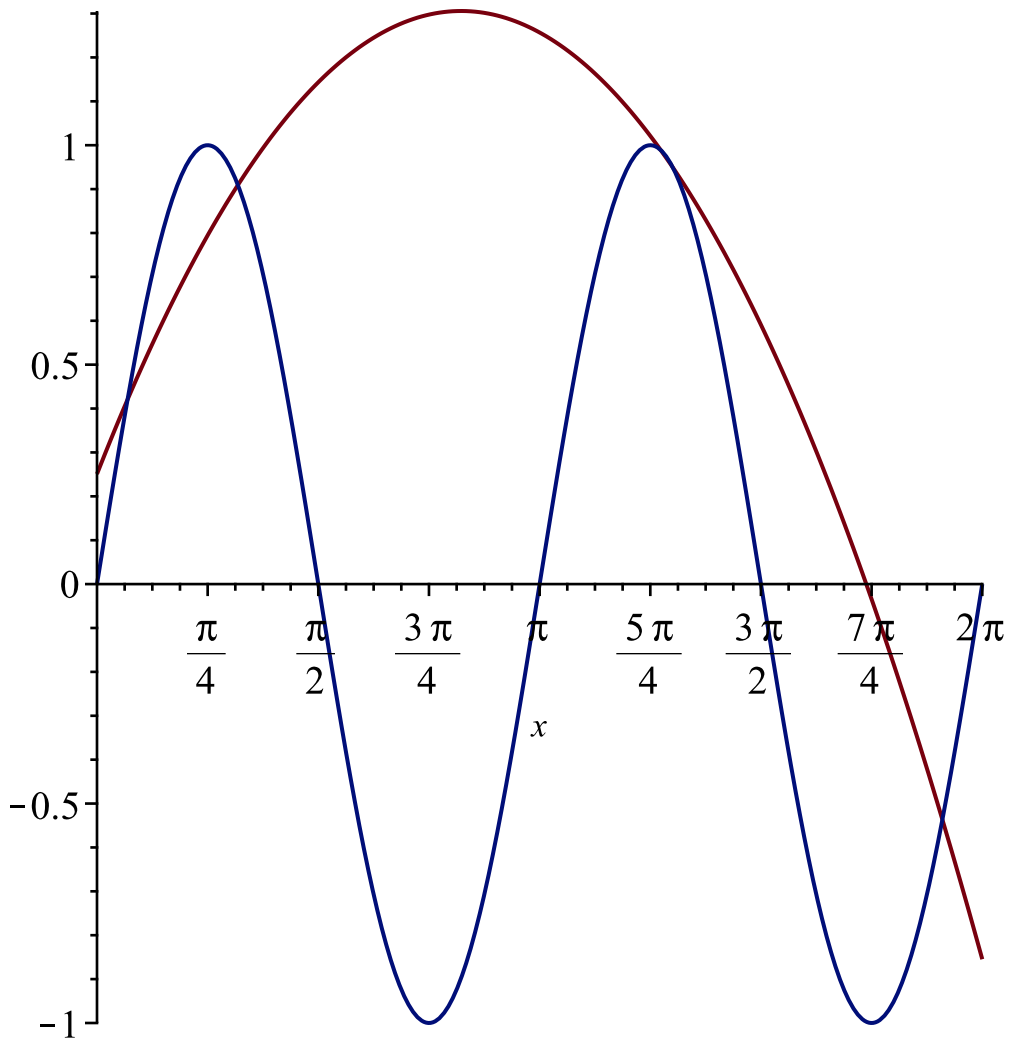
```
> f := x → sin(2 · x); fp := x → 2 · cos(2 · x);
      f := x → sin(2 x)
      fp := x → 2 cos(2 x) (1)
```

```
> f1 := f(1.); f2 := f(2.); f3 := f(3.); f4 := f(4.); f5 := f(5.); f6 := f(6.);
      fp1 := fp(1.); fp4 := fp(4.); fp6 := fp(6.);
      f1 := 0.9092974268
      f2 := -0.7568024953
      f3 := -0.2794154982
      f4 := 0.9893582466
      f5 := -0.5440211109
      f6 := -0.5365729180
      fp1 := -0.8322936730
      fp4 := -0.2910000676
      fp6 := 1.687707917 (2)
```

```
> L20 := x → (x - 4) · (x - 6) / ((1 - 4) · (1 - 6)); L21 := x → (x - 1) · (x - 6) / ((4 - 1) · (4 - 6)); L22 := x → (x - 1) · (x - 4) / ((6 - 1) · (6 - 4));
      L20 := x → 1/15 (x - 4) (x - 6)
      L21 := x → -1/6 (x - 1) (x - 6)
      L22 := x → 1/10 (x - 1) (x - 4) (3)
```

```
> L1 := x → f1 · L20(x) + f4 · L21(x) + f6 · L22(x); expand(L1(x));
      L1 := x → f1 L20(x) + f4 L21(x) + f6 L22(x)
      -0.1579305044 x2 + 0.8163394625 x + 0.2508884692 (4)
```

```
> plot( {f(x), L1(x)}, x = 0 .. 2 · Pi);
```



>  $Lp20 := \text{subs}(x=1, \text{diff}(L20(x), x)); Lp21 := \text{subs}(x=4, \text{diff}(L21(x), x)); Lp22 := \text{subs}(x=6, \text{diff}(L22(x), x));$

$$Lp20 := -\frac{8}{15}$$

$$Lp21 := -\frac{1}{6}$$

$$Lp22 := \frac{7}{10}$$

(5)

>  $H20 := x \rightarrow (1 - 2 \cdot (x - 1) \cdot Lp20) \cdot (L20(x))^2; \text{expand}(H20(x));$

$$H20 := x \rightarrow (1 - (2x - 2) Lp20) L20(x)^2 - \frac{107}{1125} x^4 + \frac{796}{1125} x^3 - \frac{7828}{3375} x^2 + \frac{3232}{1125} x - \frac{64}{375} + \frac{16}{3375} x^5$$

(6)

>  $H21 := x \rightarrow (1 - 2 \cdot (x - 4) \cdot Lp21) \cdot (L21(x))^2; H21(x);$

$$H21 := x \rightarrow (1 - (2x - 8) Lp21) L21(x)^2$$

(7)

$$\frac{1}{36} \left( -\frac{1}{3} + \frac{1}{3} x \right) (x-1)^2 (x-6)^2 \quad (7)$$

> H22 := x → (1 - 2 · (x - 6) · Lp22) · (L22(x))<sup>2</sup>; H22(x);

$$H22 := x \rightarrow (1 - (2x - 12) Lp22) L22(x)^2$$

$$\frac{1}{100} \left( \frac{47}{5} - \frac{7}{5} x \right) (x-1)^2 (x-4)^2 \quad (8)$$

> Hp20 := x → (x - 1) · (L20(x))<sup>2</sup>; Hp20(x);

$$Hp20 := x \rightarrow (x - 1) L20(x)^2$$

$$\frac{1}{225} (x-1) (x-4)^2 (x-6)^2 \quad (9)$$

> Hp21 := x → (x - 4) · (L21(x))<sup>2</sup>; Hp21(x);

$$Hp21 := x \rightarrow (x - 4) L21(x)^2$$

$$\frac{1}{36} (x-4) (x-1)^2 (x-6)^2 \quad (10)$$

> Hp22 := x → (x - 6) · (L22(x))<sup>2</sup>; Hp22(x);

$$Hp22 := x \rightarrow (x - 6) L22(x)^2$$

$$\frac{1}{100} (x-6) (x-1)^2 (x-4)^2 \quad (11)$$

> H1 := x → f1 · H20(x) + f4 · H21(x) + f6 · H22(x) + fp1 · Hp20(x) + fp4 · Hp21(x) + fp6 · Hp22(x);

$$H1 := x \rightarrow f1 H20(x) + f4 H21(x) + f6 H22(x) + fp1 Hp20(x) + fp4 Hp21(x) + fp6 Hp22(x) \quad (12)$$

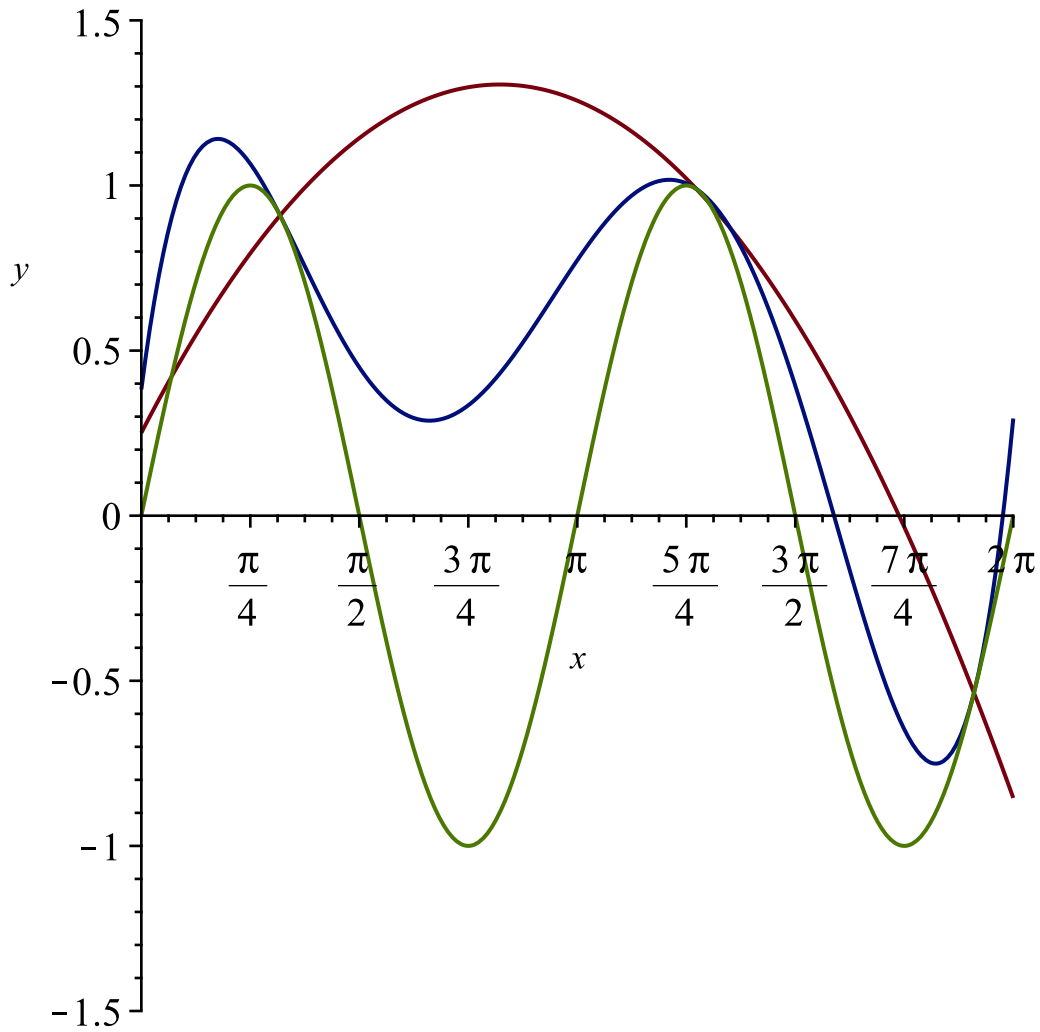
> H1(x);

$$0.004041321897 \left( -\frac{1}{15} + \frac{16}{15} x \right) (x-4)^2 (x-6)^2 + 0.02748217352 \left( -\frac{1}{3} + \frac{1}{3} x \right) (x-1)^2 (x-6)^2 - 0.005365729180 \left( \frac{47}{5} - \frac{7}{5} x \right) (x-1)^2 (x-4)^2 - 0.003699082991 (x-1) (x-4)^2 (x-6)^2 - 0.008083335211 (x-4) (x-1)^2 (x-6)^2 + 0.01687707917 (x-6) (x-1)^2 (x-4)^2 \quad (13)$$

> expand(H1(x));

$$3.256603852 x - 4.444653041 x^2 + 2.085080215 x^3 - 0.3963057091 x^4 + 0.02607814968 x^5 + 0.382493961 \quad (14)$$

> plot( {f(x), L1(x), H1(x)}, x=0 ..2·Pi, y=-1.5 ..1.5);



$$\begin{aligned}
 > L60 := x \rightarrow \frac{(x-2) \cdot (x-3) \cdot (x-4) \cdot (x-5) \cdot (x-6)}{(1-2) \cdot (1-3) \cdot (1-4) \cdot (1-5) \cdot (1-6)}; L60(x); \\
 & \quad L60 := x \rightarrow -\frac{1}{120} (x-2) (x-3) (x-4) (x-5) (x-6) \\
 & \quad \quad -\frac{1}{120} (x-2) (x-3) (x-4) (x-5) (x-6) \tag{15}
 \end{aligned}$$

$$\begin{aligned}
 > L61 := x \rightarrow \frac{(x-1) \cdot (x-3) \cdot (x-4) \cdot (x-5) \cdot (x-6)}{(2-1) \cdot (2-3) \cdot (2-4) \cdot (2-5) \cdot (2-6)}; L61(x); \\
 & \quad L61 := x \rightarrow \frac{1}{24} (x-1) (x-3) (x-4) (x-5) (x-6) \\
 & \quad \quad \frac{1}{24} (x-1) (x-3) (x-4) (x-5) (x-6) \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 > L62 := x \rightarrow \frac{(x-1) \cdot (x-2) \cdot (x-4) \cdot (x-5) \cdot (x-6)}{(3-1) \cdot (3-2) \cdot (3-4) \cdot (3-5) \cdot (3-6)}; L62(x); \\
 & \quad L62 := x \rightarrow -\frac{1}{12} (x-1) (x-2) (x-4) (x-5) (x-6) \\
 & \quad \quad -\frac{1}{12} (x-1) (x-2) (x-4) (x-5) (x-6) \tag{17}
 \end{aligned}$$

$$\begin{aligned}
> L63 := x \rightarrow \frac{(x-1) \cdot (x-2) \cdot (x-3) \cdot (x-5) \cdot (x-6)}{(4-1) \cdot (4-2) \cdot (4-3) \cdot (4-5) \cdot (4-6)}; L63(x); \\
L63 := x \rightarrow \frac{1}{12} (x-1) (x-2) (x-3) (x-5) (x-6) \\
\frac{1}{12} (x-1) (x-2) (x-3) (x-5) (x-6) \tag{18}
\end{aligned}$$

$$\begin{aligned}
> L64 := x \rightarrow \frac{(x-1) \cdot (x-2) \cdot (x-3) \cdot (x-4) \cdot (x-6)}{(5-1) \cdot (5-2) \cdot (5-3) \cdot (5-4) \cdot (5-6)}; L64(x); \\
L64 := x \rightarrow -\frac{1}{24} (x-1) (x-2) (x-3) (x-4) (x-6) \\
-\frac{1}{24} (x-1) (x-2) (x-3) (x-4) (x-6) \tag{19}
\end{aligned}$$

$$\begin{aligned}
> L65 := x \rightarrow \frac{(x-1) \cdot (x-2) \cdot (x-3) \cdot (x-4) \cdot (x-5)}{(6-1) \cdot (6-2) \cdot (6-3) \cdot (6-4) \cdot (6-5)}; L65(x); \\
L65 := x \rightarrow \frac{1}{120} (x-1) (x-2) (x-3) (x-4) (x-5) \\
\frac{1}{120} (x-1) (x-2) (x-3) (x-4) (x-5) \tag{20}
\end{aligned}$$

$$\begin{aligned}
> L6 := x \rightarrow f1 \cdot L60(x) + f2 \cdot L61(x) + f3 \cdot L62(x) + f4 \cdot L63(x) + f5 \cdot L64(x) + f6 \cdot L65(x); \\
\text{expand}(L6(x)); \\
L6 := x \rightarrow f1 L60(x) + f2 L61(x) + f3 L62(x) + f4 L63(x) + f5 L64(x) + f6 L65(x) \\
20.54916114 x - 19.92859790 x^2 - 6.348415418 + 7.917971634 x^3 - 1.365638342 x^4 \\
+ 0.08481633485 x^5 \tag{21}
\end{aligned}$$

$$> \text{plot}(\{f(x), L1(x), H1(x), L6(x)\}, x=0..2 \cdot \text{Pi}, y=-2..1.5);$$

