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This Worksheet demonstrates commands for Laplace Transforms.
We use this to solve the following initial value problem.

> $de := diff(y(t), t\$2) + 4 \cdot diff(y(t), t) + 13 \cdot y(t) = 36 \cdot t \cdot \exp(-2 \cdot t) \cdot \sin(3 \cdot t);$

$$de := \frac{d^2}{dt^2}\, y(t) + 4\, \frac{d}{dt}\, y(t) + 13\, y(t) = 36\, t\, e^{-2\,t} \sin(3\, t) \tag{1}$$

The initial conditions are

> $y(0) := -3;\ D(y)(0) := 6;$

$$y(0) := -3$$
$$D(y)(0) := 6 \tag{2}$$

We need Maple's integral transform package

> $with(inttrans):$

Before solving our problem above, we demonstrate some basic features such as performing a Partial Fractions Decomposition (PFD).

> $F := s \rightarrow \frac{(3 \cdot s^2 + 5 \cdot s - 12)}{(s^3 - s^2 - 6 \cdot s) \cdot (s^2 + 4 \cdot s + 5)};$

$$F := s \mapsto \frac{3\, s^2 + 5\, s - 12}{(s^3 - s^2 - 6\, s)\, (s^2 + 4\, s + 5)} \tag{3}$$

> $convert(F(s), parfrac, s);$

$$\frac{34\, s - 9}{65\, (s^2 + 4\, s + 5)} + \frac{1}{13\, (s - 3)} + \frac{2}{5\, s} - \frac{1}{s + 2} \tag{4}$$

Command for finding the Laplace transform of a function. (One not readily in our Table.)

> $laplace(t \cdot \exp(-2 \cdot t) \cdot \sin(3 \cdot t), t, s);$

$$\frac{6\, (s + 2)}{((s + 2)^2 + 9)^2} \tag{5}$$

Command for finding the Inverse Laplace transform.

> $invlaplace\left(\frac{18}{((s + 5)^2 + 9)^2}, s, t\right);$

$$\frac{e^{-5\,t}\, (\sin(3\, t) - 3\, t \cos(3\, t))}{3} \tag{6}$$

We proceed with a series of commands to solve the original IVP.
First taking the Laplace transform of the differential equation.

> $soln := laplace(de, t, s);$

$$soln := s^2\, laplace(y(t), t, s) + 6 + 3\, s + 4\, s\, laplace(y(t), t, s) + 13\, laplace(y(t), t, s) \tag{7}$$

$$= \frac{216\,(s+2)}{\left((s+2)^2+9\right)^2}$$

Next use Maple's algebra to find Y(s).

> *soln1* := *solve*(*soln*, *laplace*(*y*(*t*), *t*, *s*));

$$soln1 := -\frac{3\,(s^5+10\,s^4+58\,s^3+188\,s^2+305\,s+194)}{(s^2+4\,s+13)^3} \quad \textbf{(8)}$$

Perform a PFD.

> *soln2* := *convert*(*soln1*, *parfrac*, *s*);

$$soln2 := \frac{216\,s+432}{(s^2+4\,s+13)^3} + \frac{-3\,s-6}{s^2+4\,s+13} \quad \textbf{(9)}$$

Take inverse Laplace transform to obtain solution.

> *invlaplace*(*soln2*, *s*, *t*);

$$e^{-2\,t}\left(t\sin(3\,t) - 3\cos(3\,t)\,(t^2+1)\right) \quad \textbf{(10)}$$

Standard method of solving the IVP follows with its graph (after clearing ICs from above).

> *y*(0) :='*y*(0)'; D(*y*)(0) :='D(*y*)(0)';

$$y(0) := y(0)$$
$$D(y)(0) := D(y)(0) \quad \textbf{(11)}$$

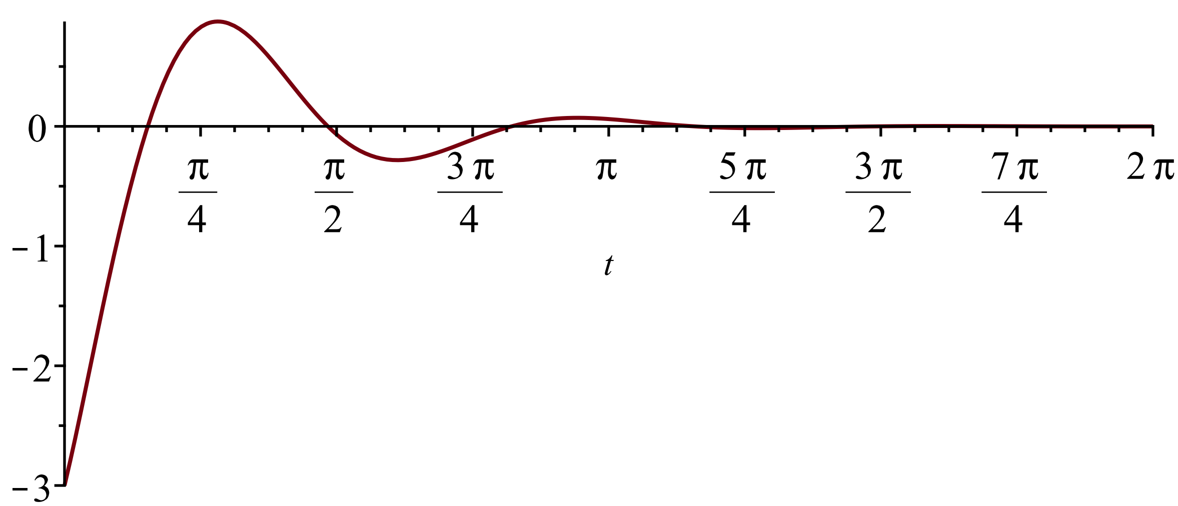
> *dsolve*({*de*, *y*(0) =−3, D(*y*)(0) = 6}, *y*(*t*));

$$y(t) = -3\,e^{-2\,t}\cos(3\,t) - 3\,t\,e^{-2\,t}\left(t\cos(3\,t) - \frac{\sin(3\,t)}{3}\right) \quad \textbf{(12)}$$

> *z* := *unapply*(*rhs*(%), *t*);

$$z := t \mapsto -3\,e^{-2\,t}\cos(3\,t) - 3\,t\,e^{-2\,t}\left(t\cos(3\,t) - \frac{\sin(3\,t)}{3}\right) \quad \textbf{(13)}$$

> *plot*(*z*(*t*), *t* = 0 ..2·Pi);



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