1. **(1 pt) mathbioLibrary/setABiocLabs/Lab121_L4_beetle.pg**
Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

One form of model that is commonly used in population dynamics is the discrete dynamical model, which has the general form:

$$P_{n+1} = F(P_n)$$

for some function, $F(P)$. This function is called the **updating function**, which we will study in more detail later. The model states that the population at the next time step is some function of the population at the current time. If the updating function is linear, we obtain a Malthusian growth model. One of the most popular forms is a quadratic or logistic model, where the linear part gives Malthusian growth and the quadratic part is negative due to crowding effects on the population. There are numerous other types of updating functions that ecologists use for $F(P)$ when they are modeling. This lab will study three different updating functions: Logistic, Beverton-Holt, and Ricker's. These will all be fit to an experimental study of beetles grown on a restricted diet.

A. C. Crombie [1] studied *Oryzaephilus surinamensis* , the saw-tooth grain beetle, with an almost constant nutrient supply (maintained 10 g. of cracked wheat weekly). The data below show the adult population of *Oryzaephilus* from Crombie's study (with some minor modifications to fill in uncollected data, provide an initial shift of one week, and give slightly different versions for different students).

Week	Adults	Week	Adults
0	3	16	400
2	6	18	467
4	23	20	428
6	61	22	434
8	145	24	424
10	281	26	472
12	346	28	435
14	363	30	471

a. The discrete logistic growth model for the adult population P_n can be written

$$P_{n+1} = f(P_n) = rP_n - mP_n^2,$$

where the constants r and m must be determined from the data.

Begin by plotting P_{n+1} vs. P_n, which you can do by entering the adult population data from times 2-30 for P_{n+1} and times 0-28 for P_n. (Be sure that P_n is on the horizontal axis.) To find the appropriate constants use Excel's trendline with its polynomial fit of order 2 and with the intercept set to 0 (under options). Give the best values of the constants, r and m, and write the equation of the model, which fits the data best. Also, find the sum of square errors between the data and this model.

$r =$ _______
$m =$ _______
$f(P) =$ ________________
$SSE =$ _______

Find all equilibria for this model (with $P_{1e} < P_{2e}$). Note that equilibria are found by solving

$$P_e = f(P_e)$$

The largest equilibrium represents the carrying capacity of the population, which is limited by the cracked wheat supply.
$P_{1e} =$ _____
$P_{2e} =$ _____
Find the derivative of the logistic updating function, then find the value of the derivative at all equilibria and give the stability of the equilibria. An equilibrium is stable if the absolute value of the derivative at the equilibrium is less than 1 and unstable if the absolute value is greater than 1.
$f'(P) =$ ____________________
$f'(P_{1e}) =$ _____ STABLE or UNSTABLE _____
$f'(P_{2e}) =$ _____ STABLE or UNSTABLE _____

b. Another important model used for population dynamics is the Beverton-Holt model, which is given by

$$P_{n+1} = B(P_n) = \frac{aP_n}{1 + \frac{P_n}{b}},$$

where the constants a and b must be determined from the data. Give the best values of the constants, a and b, and write the equation of the model, which fits the data best. (You can make an initial guess of $a = 3$ and $b = 200$.) Also, find the sum of square errors between the data and this model.
$a =$ _______
$b =$ _______
$B(P) =$ ________________
$SSE =$ _______

Find all equilibria for this model (with $P_{1e} < P_{2e}$). Note that equilibria are found by solving

$$P_e = B(P_e)$$

The largest equilibrium represents the carrying capacity of the population.
$P_{1e} =$ _____
$P_{2e} =$ _____
Find the derivative of the Beverton-Holt updating function, then find the value of the derivative at all equilibria and give the stability of the equilibria.
$B'(P) =$ ____________________
$B'(P_{1e}) =$ _____ STABLE or UNSTABLE _____
$B'(P_{2e}) =$ _____ STABLE or UNSTABLE _____

c. Another model used for population dynamics is Ricker's model, which is more often used for fish populations and is given by

$$P_{n+1} = R(P_n) = aP_n e^{-\frac{P_n}{b}},$$

where the constants a and b must be determined from the data. Give the best values of the constants, a and b, and write the equation of the model, which fits the data best. (You can make an initial guess of $a = 2.5$ and $b = 500$.) Also, find the sum of square errors between the data and this model.

$a =$ ______
$b =$ ______
$R(P) =$ ______________
$SSE =$ ______

Find all equilibria for this model (with $P_{1e} < P_{2e}$). Note that equilibria are found by solving

$$P_e = R(P_e)$$

The largest equilibrium represents the carrying capacity of the population.
$P_{1e} =$ ____
$P_{2e} =$ ____
Find the derivative of the Ricker's updating function, then find the value of the derivative at all equilibria and give the stability of the equilibria.
$R'(P) =$ ______________
$R'(P_{1e}) =$ ____ STABLE or UNSTABLE ____
$R'(P_{2e}) =$ ____ STABLE or UNSTABLE ____

d. In this part of the problem we use your techniques from this course to find all intercepts, horizontal asymptotes, critical points (maxima), and points of inflection for all three updating functions on their domains, which is for $P \geq 0$. Use the parameters found above to determine the values for each of these graphical features and insert the word **NONE** if the feature doesn't exist for the particular function.

Begin by considering the updating function for the logistic growth model, $f(P)$. Find the x-intercepts ($x_1 < x_2$), y-intercept, x and y values of the maximum, (x_c, y_c), y of a horizontal asymptote, and x and y values of any points of inflection (with $x > 0$), (x_p, y_p).
x-intercepts: $x_1 =$ ____ $x_2 =$ ____
y-intercept = ____
Maximum: $x_c =$ ____ $y_c =$ ____
Horizontal asymptote $y =$ ____
Point of inflection: $x_p =$ ____ $y_p =$ ____

Next consider the updating function for the Beverton-Holt growth model, $B(P)$. Find the x-intercept, y-intercept, x and y values of the maximum, (x_c, y_c), y of a horizontal asymptote, and x and y values of any points of inflection (with $x > 0$), (x_p, y_p).
x-intercept = ____
y-intercept = ____
Maximum: $x_c =$ ____ $y_c =$ ____
Horizontal asymptote $y =$ ____
Point of inflection: $x_p =$ ____ $y_p =$ ____

Finally consider the updating function for the Ricker's growth model, $R(P)$. Find the x-intercept, y-intercept, x and y values of the maximum, (x_c, y_c), y of a horizontal asymptote, and x and y values of any points of inflection (with $x > 0$), (x_p, y_p).
x-intercept = ____
y-intercept = ____
Maximum: $x_c =$ ____ $y_c =$ ____
Horizontal asymptote $y =$ ____
Point of inflection: $x_p =$ ____ $y_p =$ ____

e. In your Lab Report, graph all three updating functions 1. logistic growth, $f(P)$ 2. Beverton-Holt function, $B(P)$ 3. Ricker's function, $R(P)$. Include the original data in your graph and add the identity map,

$$P_{n+1} = P_n.$$

(All of these functions are to be on a single graph and labeled properly. Take your domain to be approximately twice the value of the largest equilibrium and take the range to be about 1.5 times the largest equilibrium.)

Discuss the similarities and differences that you observe between the three models. Compare the models to the experimental data. Which model appears to fit the data best? Compare the values and stability of the equilibria for each of the models. How does this match the data at large times? Find the equilibria on the graph and relate this to the identity map. Which updating function makes the most sense based on your knowledge of populations? Explain your reasoning.

f. The discrete population models are given by the equations:

$$p_{n+1} = f(p_n),$$

$$p_{n+1} = B(p_n),$$

$$p_{n+1} = R(p_n),$$

where the functions are given above for each model and the best fitting parameters have been found. In this part of the problem, you simulate each of the models with the discrete dynamical models, and use Excel's Solver to best fit the initial value, p_0. This process creates the times series simulation of the data. As an initial guess, start with your initial population as ($p_0 = 3$) starting at $t = 0$ and simulate the growth for 30 weeks. Use Excel's Solver to find the best possible p_0 value that minimizes the square error between the populations in the data and the populations given by each of the models. (You are performing 3 different simulations for the 3 models.) Give this best p_0 values for each model and list the populations at times $t = 6$, 12, and 24. Give the percent error between the models and the data at these times. Also, include the value of the least sum of square errors between your simulations and the data.

For the logistic growth model:

$P_0 =$ _____

Sum of Square Errors = _____

List the beetle population at times $t = 6$, 12, and 24 weeks.

$P_6 =$ _____ Percent error = _____

$P_{12} =$ _____ Percent error = _____

$P_{24} =$ _____ Percent error = _____

For the Beverton-Holt growth model:

$P_0 =$ _____

Sum of Square Errors = _____

List the beetle population at times $t = 6$, 12, and 24 weeks.

$P_6 =$ _____ Percent error = _____

$P_{12} =$ _____ Percent error = _____

$P_{24} =$ _____ Percent error = _____

For the Ricker's growth model:

$P_0 =$ _____

Sum of Square Errors = _____

List the beetle population at times $t = 6$, 12, and 24 weeks.

$P_6 =$ _____ Percent error = _____

$P_{12} =$ _____ Percent error = _____

$P_{24} =$ _____ Percent error = _____

g. In your Lab Report, create a single graph that contains all three model simulations and the time-series data. Describe how well the models fit the actual data, and determine which model best matches the actual data. Compare by using your least sum of square errors and percent errors that you computed. Which model best fits the actual initial starting population? Write a short discussion that compares and contrasts the three models. Which model do you believe is better and why? Do these models differ significantly in what they predict as the carrying capacity of the beetle population under these experimental conditions.

[1] A. C. Crombie (1946) On competition between different species of graminivorous insects, Proc. R. Soc. (B), 132, 362-395.