Radioactive isotopes are very important in a number of medical applications. Most of these isotopes are created in nuclear reactors, then processed and delivered to hospitals for various applications. These isotopes are often very short-lived and must be shipped weekly for medical procedures. Molybdenum-99 (Mo-99) is an isotope that results from the fission of U-235 in nuclear reactors (and atomic bombs). It is relatively short-lived, and it decays into Technetium-99, which has a very short half-life. The Technetium-99 (Tc-99) produces very high-quality medical images for cancers, heart disease, and other serious medical problems. Unfortunately, many of the nuclear reactors licensed to produce Mo-99 (none in the U.S.) are aging and often need to shut down for repairs. Recently, this resulted in a 70 percent drop in world supplies [1], which meant that significant medical procedures had to be delayed or cancelled. In this problem, we use the nuclear chemistry of Mo-99 and Tc-99 to determine half-lifes and show how to calculate supplies.

a. Below is a table showing the amount of Mo-99 and Tc-99 at various times.

<table>
<thead>
<tr>
<th>( t ) (hr)</th>
<th>( M(t) )</th>
<th>( W(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19.64</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>18.55</td>
<td>74.7</td>
</tr>
<tr>
<td>10</td>
<td>17.59</td>
<td>109</td>
</tr>
<tr>
<td>20</td>
<td>15.82</td>
<td>134.8</td>
</tr>
<tr>
<td>40</td>
<td>12.89</td>
<td>124.4</td>
</tr>
<tr>
<td>60</td>
<td>10.43</td>
<td>103</td>
</tr>
<tr>
<td>80</td>
<td>8.42</td>
<td>81.1</td>
</tr>
</tbody>
</table>

Mo-99 decays with a characteristic half-life according to the equation:

\[
M(t) = M_0 e^{-r_1 t},
\]

where \( t \) is in hours. Use the data on Mo-99 and Excel’s Trendline with an exponential fit to find the best parameters, \( M_0 \) and \( r_1 \). Also, compute the sum of square errors between the model and the data.

\( M_0 = \) __________

\( r_1 = \) __________

\( SSE = \) __________

Use this information to determine the half-life of Mo-99.

Half-Life = ________ hrs

Find the \( M \)-intercept and the horizontal asymptote for this model.

\( M \)-intercept = ________

Horizontal Asymptote \( M = \) ________

b. In your Lab report, create a graph with the data and the model for the radioactive decay of Mo-99 for \( t \in [0, 100] \). Create a short paragraph that briefly describes how well the model simulates the data.

c. Molybdenum-99 undergoes a gamma decay to produce the radioactive element Technetium-99, which is the primary isotope that is used for the medical images. The equation that describes the amount of Tc-99 coming from the Mo-99 is given by

\[
W(t) = W_0 (e^{-r_1 t} - e^{-r_2 t}),
\]

where \( r_1 \) comes from your calculation above. Find the least squares best fit of this model to the data in the Table above, using Excel’s Solver to find the best parameters, \( W_0 \) and \( r_2 \). (For starting values, take \( W_0 = 200 \) and \( r_2 = 0.1 \).) Also, compute the sum of square errors between the model and the data.

\( W_0 = \) ________

\( r_2 = \) ________

\( W(t) = \) __________

\( SSE = \) __________

The decay constant \( r_2 \) gives the decay rate of Tc-99. Use this information to determine the half-life of Tc-99.

Half-Life = ________ hrs

Find the \( W \)-intercept and the horizontal asymptote for this model.

\( W \)-intercept = ________

Horizontal Asymptote \( W = \) ________

The rate of radioactive decay, which determines the amount of radioactivity from Mo-99, is given by the derivative of \( M(t) \). Find this derivative.

\( M'(t) = \) __________

Find the time, \( t_{\text{max}} \), where the maximum amount of Tc-99 occurs and the value of \( W(t_{\text{max}}) \).

\( t_{\text{max}} = \) ________ hrs

\( W(t_{\text{max}}) = \) ________ g.

After the maximum occurs, the amount of Tc-99 first decreases more and more rapidly, then after a point of inflection this rate of radioactive decay decreases. Find the second derivative of \( W(t) \).

\( W''(t) = \) __________

Find the point of inflection for \( W(t) \).

Time for point of inflection \( t_i = \) ________ hrs.

Determine both the amount of Tc-99 at that time and the rate of radioactive decay at that time, i.e., find

\( W(t_i) = \) ________ g.

\( W'(t_i) = \) ________ g/hr.
d. In your Lab report, create a graph with the data and the model for the radioactive decay of Tc-99 for \( t \in [0, 100] \). Create a short paragraph that briefly describes how well the model simulates the data. Use the model above to help explain why hospitals need to have weekly shipments for their medical imaging.