

**1. (1 pt) mathbioLibrary/setABiocLabs/Lab121\_F4\_nonauto.pg**

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on all problems.

Using data from the U. S. census bureau, the table below presents the population (in millions) for Mexico. Human populations tend to have growth rates vary with time because of advancement of living conditions from technology. Below we develop a nonautonomous model for the population growth of Mexico based on the following data.

Year	Population	Year	Population
1950	28.51	1980	68.33
1960	38.57	1990	84.9
1970	52.77	2000	99.93

a. We begin our modeling effort by finding the growth rate of Mexico for each decade. Let 1950 correspond to  $t = 0$ . Find the growth rate for each decade using the data above by dividing the population from one decade by the population of the previous decade and subtracting 1 from this ratio. Associate each growth rate with the earlier value of  $t$  from the two census dates. Find the growth rate at  $t = 30$ .

Growth at  $t = 30$  is \_\_\_\_\_

Determine the average (mean) growth rate,  $r$ , (per decade) from the data above.

Average growth rate  $r =$  \_\_\_\_\_

Create a graph of the growth rates for each decade and their corresponding  $t$  values with  $t = 0, 10, 20, 30, 40$ . Use EXCEL's trendline to find the best straight line

$$k(t) = a + bt$$

through the growth data. This equation has parameters

$a =$  \_\_\_\_\_ and  $b =$  \_\_\_\_\_

Producing the best fitting line

$k(t) =$  \_\_\_\_\_

Assuming that the data produces the best value, find the percent error between the model growth rate,  $k(t)$ , and the actual growth rate at  $t = 30$ .

Percent Error = \_\_\_\_\_

b. In your Lab Report, plot the growth data for Mexico as a function of  $t$  (years after 1950). Include on this graph a line representing the average growth rate,  $r$ , over the 50 years of data and the best fitting line through the data,  $k(t)$ . Briefly discuss what you observe about the growth rate for Mexico. How well does  $k(t)$  fit the growth rate data?

c. The Discrete Malthusian growth model is given by

$$P_{n+1} = (1 + r)P_n, \quad P_0 = 28.51.$$

where  $r$  is the average growth rate (computed above) and  $P_0$  is the population in 1950  $t = 0$ . Write the general solution to this

model, where  $n$  is in decades. Your answer should include the values for  $r$  and  $P_0$ , but depend on the variable  $n$ .

$P_n =$  \_\_\_\_\_

Find the sum of square errors between this discrete Malthusian growth model and the data in the Table.

Sum of Square Errors for discrete Malthusian model = \_\_\_\_\_

Use this model to estimate the populations in 1960, 1980, and 2000. Then with the actual population being accurate, find the percent error from this Malthusian growth model

$P_1 =$  \_\_\_\_\_ Percent error = \_\_\_\_\_

$P_3 =$  \_\_\_\_\_ Percent error = \_\_\_\_\_

$P_5 =$  \_\_\_\_\_ Percent error = \_\_\_\_\_

Use the model to predict the population in 2020 and 2050.

$P_7 =$  \_\_\_\_\_

$P_{10} =$  \_\_\_\_\_

c. The nonautonomous Malthusian growth model is given by

$$P_{n+1} = (1 + k(t_n))P_n, \quad P_0 = 28.51.$$

where  $k(t_n)$  is computed above (with  $t_n = 10n$ ) and  $P_0$  is again the population in 1950. Simulate this nonautonomous discrete dynamical model from 1950 to 2050 ( $t_n \in [0, 100]$ ). Find the sum of square errors between this nonautonomous Malthusian growth model and the data in the Table.

Sum of Square Errors for nonautonomous Malthusian model = \_\_\_\_\_

Use this model to estimate the populations in 1960, 1980, and 2000. Then with the actual population being accurate, find the percent error from this nonautonomous Malthusian growth model

$P_1 =$  \_\_\_\_\_ Percent error = \_\_\_\_\_

$P_3 =$  \_\_\_\_\_ Percent error = \_\_\_\_\_

$P_5 =$  \_\_\_\_\_ Percent error = \_\_\_\_\_

Use the model to predict the population in 2020 and 2050.

$P_7 =$  \_\_\_\_\_

$P_{10} =$  \_\_\_\_\_

d. The nonautonomous Malthusian growth model has a declining linear growth rate. Hence, its growth rate will eventually go to zero, then become negative. According to this model, the population of Mexico achieves a maximum during the decade following the time when the growth rate is zero. Use the growth rate function  $k(t)$  to predict when the growth rate becomes zero.  $k(t_M) = 0$  when  $t_M =$  \_\_\_\_\_

Simulate the nonautonomous Malthusian growth model until it achieves a maximum, and list the maximum value from the simulation along with the value of  $t_n$  when that maximum occurs.

Maximum population = \_\_\_\_\_ with  $t_n =$  \_\_\_\_\_

e. In your Lab Report, create a graph of the population data for Mexico. Show the Malthusian growth and the nonautonomous Malthusian growth models on the this graph with modeling projections for the period from 1950 to 2050. Briefly discuss how well these models predict the population over this

period. How believable are the modeling projections for the future? List some strengths and weaknesses of each of the models and how you might obtain a better means of predicting the future population of Mexico.