1. **(20 pts) mathbioLibrary/setABiocLabs/Lab121_E3_allegheny.pg**
Because of the accuracy of WebWork, you should use 5 or 6 significant figures on all problems.

The data below came from the Allegheny National Forest in Pennsylvania. The issue was whether either the diameter or the height of a tree accurately predicts the volume of wood in the tree. Using the data below, you are to see if there exists a meaningful relation between these variables. Thus, you want to find the volume as a function of either diameter or height. The volume is measured in board feet.

Diameter (in)	Height (ft)	Volume (board ft)
8.4	62	10.5
10.8	83	18.8
11.3	72	18.2
11.4	78	21.5
12.1	72	18.8
13.1	88	27.3
14.2	72	36.3
16.3	69	38.7
17.2	81	55.2
18.2	78	51.9
20.7	87	77.1

a. The reference for this model suggests a simple linear model, so use Excel's trendline to find the best line through the data. Begin with a linear model for the volume, V_d, in board ft as a function of the diameter, d, in inches

$$V_d = md + b.$$

The best linear model has a slope, m, and intercept, b with

$m =$ _____
$b =$ _____

Determine the sum of square errors between this model and the data.

Sum of Square Errors = _____.

Repeat this process for a linear model for the volume, V_h, in board ft as a function of the height, h, in ft

$$V_h = kh + c.$$

This best linear model has a slope, k, and intercept, c with

$k =$ _____
$c =$ _____

Determine the sum of square errors between this model and the data.

Sum of Square Errors = _____.

Consider the tree that has a volume of 21.5 board ft. The data give this tree a diameter of 11.4 in and a height of 78 ft. Use the models above to predict the expected volume and the percent error for each of the models (assuming the measured value to be the best value).

For $d = 11.4$, $V_d =$ _____
with percent error = _____.

For $h = 78$, $V_h =$ _____
with percent error = _____.

Based on the two linear models and the graphs with the data, which data set (DIAMETER or HEIGHT) better predicts the volume of a tree? _____

b. In your Lab report, create a graph of the data and a linear model for volume as a function of diameter. Show the formula for the model on the graph. Also, create a second graph of the data and a linear model for volume as a function of height with its formula on the graph. Write a brief explanation for why you would expect one of these measurements to better predict the volume based on the biology of trees. What happens with both models as the diameter or height gets close to zero? Is this biologically feasible? Interpret what the model is predicting from a biological perspective. Can you describe adjustments to the model that might work for small diameters?

c. For this part of the problem, we examine an allometric model for the volume as a function of the diameter. Consider the allometric model given by

$$V = Kd^A.$$

The best fitting coefficients, K and A, are given by

$K =$ _____.
$A =$ _____.

Determine the sum of square errors between the weight model and the data.

Sum of Square Errors = _____.

Consider the tree that has a volume of 21.5 board ft. The data

give this tree a diameter of 11.4 in. Use this model to predict the expected volume and the percent error for the model (assuming the measured value to be the best value).

For $d = 11.4$, $V =$ _____
with percent error = _____.

d. In your Lab Report, plot the data and the best allometric model. Be sure to include the formula for the model on your graph. What is the domain and range of this allometric model (mathematically)? From a biological perspective, what is the domain and range for the allometric model? Find a reasonable explanation for the power that you have obtained with the allometric model. (You want to round the power to the nearest integer and explain why that integer value is relevant to relating volume of a tree to it diameter to this power.) Your discussion should include reasons why the tree becomes larger in its trunk in order to survive.

e. One method (the one used by Excel) of fitting an allometric or power law model to a set of data is to find the best straight line through the logarithms of the data. In the case above, we have an allometric model of the form $V = Kd^A$, which can be written in the linear logarithmic form

$$\ln(V) = \ln(K) + A\ln(d).$$

In the table above, take the logarithm of the volume ($\ln(V)$) and the logarithm of the diameter($\ln(d)$). Graph the ($\ln(V)$) vs ($\ln(d)$) with Excel's scatter plot, then apply a linear fit under trendline to see how this fits the data.

What is the value of the slope of this best fitting line?
Slope = _____.

What is the value of the $\ln(V)$-intercept?
Intercept = _____.

f. In your Lab Report, detail the mathematical steps and use the Equation Editor in Word to transform the allometric model to the linear logarithmic form. Plot the logarithm of the data and the best straight line fit to these data. Show the formula for the best fitting linear model on the graph. How well does the graph match the data? Write a brief discussion of how the coefficients obtained in this manner compare to the ones found in Part c. For this logarithmic form of the model, find the domain and range of the graph (mathematically). Again use your biological knowledge to give limits on the domain and range of the logarithmic model. Also, have Excel plot a log-log plot of the data and the trendline that you found in Part c. (This is done by editing the graph and selecting logarithmic scales for both the x and y axes, which is easily done by double clicking on the axes.) Do the data roughly fall on a straight line in this log-log plot? Write a brief discussion comparing these different forms of the allometric model graphs shown in Parts d and f. Discuss the similarities and differences between these two graphs.