1. **(20 pts) mathbioLibrary/setABioc2Labs/Lab122_B5_daylight_temp.pg** Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

This Problem is set for 2009.

In the Fall we see the days getting shorter and cooler, while in the Spring we see the days getting longer and warmer. In the first part of this problem we will model the length of the day in minutes, $L(t)$, as a function of the date, t, using the sine function. The model to be considered is

$$L(t) = a + b\sin(\omega(t-\phi)),$$

where the constants a, b, ω, and ϕ must be determined from data about the length of the day in **San Diego in 2009.** (The constants are positive and $0 \le \phi < 365$.) You will use the information from the Navy website:

http://www.usno.navy.mil/USNO/astronomical-applications/data-services

to obtain the data you need to solve this problem. This problem is designed based on the data from the table:

Table of Sunrise/Sunset, Moonrise/Moonset, or Twilight Times for an Entire Year

Note there is a table on daylight duration, which because of rounding may give slightly different answers; however, can be used as a check.

a. First find the longest and shortest days from the Navy website by searching for the Summer and Winter Solstices.
Summer Solstice is June ______
Winter Solstice is December ______
Then use the website to generate a Table of Sunrise/Sunset data for San Diego and convert the length of the longest and shortest days to minutes.

Longest Day = ______ min
Shortest Day = ______ min

The parameter a represents the average length of day for the year. The parameter b is the amplitude for the variation of the length of day. The parameter ϕ is the phase shift to adjust the sine function's maximum to match the longest day of the year. The parameter ω relates to the period of this function. Most years are 365 days long, but we have leap years (366 days) for most years that are divisible by 4. Pope Gregory XIII noticed that Easter was getting earlier over the centuries, so with the help of Christopher Clavius, they revised the calendar. What is the next year that is divisible by 4 that is only 365 days long? (Hint: This question is asking about the calendar that we use, not mathematical.)

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In this problem, we use the actual length of the calendar year to 7 significant figures, which is 365.2425. Use this accurate value of the calendar year to compute ω.

List the values of your parameters:

$a =$ ______
$b =$ ______
$\phi =$ ______
$\omega =$ ______

It follows that we can write our length of day function as:
$L(t) =$ ______________________ min

b. Let $t = 0$ correspond to January 1. Use this model to estimate the length of day on Memorial Day (5/25), then find the actual length of the day from the website listed above and determine the percent error between the model and the Navy tables. (Assume that the value from the Navy tables is the best value.)

Model Length of Day = ______
Percent Error = ______ Repeat this process for Thanksgiving (11/26), finding the length of day according to the model and the percent error of the model prediction as compared to the actual value from the Navy tables.

Model Length of Day = ______
Percent Error = ______

c. We would like to know the rate at which the daylight is changing for San Diego in min/day. A good estimate of this uses the model above to compute:

$$\Delta L(t) = L(t+1) - L(t).$$

Find the value of t when this rate of change is increasing most rapidly and what that rate of change in daylight is

$t_{max} =$ ______ day
$\Delta L(t_{max}) =$ ______ min/day

Also, find the value of t when this rate of change is decreasing most rapidly and what that rate of change in daylight is

$t_{min} =$ ______ day
$\Delta L(t_{min}) =$ ______ min/day

Find the change in daylight for Memorial Day (5/25).
Rate of Change = ______ min/day
Find the change in daylight for Thanksgiving (11/26).
Rate of Change = ______ min/day

d. In your Lab report, create a graph of $L(t)$. Use your error analysis to discuss how well this model fits the data. Write a brief paragraph describing the shape of the graph and how different features on the graph relate to different times of the year. What times of the year is the model closest to the Navy tables and when does the model deviate most from the data? Explain why you might expect this.

Create a graph of the change in daylight, $\Delta L(t)$. Write a paragraph briefly describing the shape of this curve and give some observations about how this graph relates to the graph of $L(t)$. Determine the time of year when daylight is increasing the most and when it is decreasing the most. What calendar events correspond to these times? What times of year is the change in daylight closest to zero? What calendar events correspond to these times?

e. Below is a table of the average high temperature for San Diego in 2009.

Month	Temp	Month	Temp	Month	Temp
Jan	70	May	68	Sep	78
Feb	65	Jun	70	Oct	72
Mar	66	Jul	75	Nov	69
Apr	68	Aug	78	Dec	63

Let each of these entries correspond to the 15th of each month (except use Feb 14). Again let $t = 0$ correspond to January 1, so for example we have May 15 is $t = 134$. We want to use these data to find the best fitting model for temperature for San Diego in 2009. The model to be considered is

$$T(t) = A + B\sin(\omega(t-\psi)),$$

where the constants A, B, ω, and ψ must be determined from data. (The constants are positive and $0 \le \psi < 365$.) Since we are on an annual cycle again, use the same value of ω as before. However, this time we use Excel's Solver to find the nonlinear least squares best fit to the data. This directly finds the best fitting parameters A, B, and ψ. The values of your parameters:

$A =$ ________
$B =$ ________
$\psi =$ ________

The least sum of square errors is given by
SSE = ________

It follows that we can write our average high temperature function as:
$T(t) =$ ____________________________ °F.

f. Again let $t = 0$ correspond to January 1. Use this temperature model to estimate the temperature on Memorial Day (5/25) and determine the percent error between the model and the actual value, 65, given by the **Weather Underground**. (Assume that the value from the Weather Underground is the best value.). If we define the average daily rate of change in temperature as:

$$\Delta T(t) = T(t+1) - T(t),$$

then also compute the average daily rate of change in temperature on Memorial Day (5/25).
Model Temperature of Day = ________
Percent Error = ________ Rate of Change of Temperature on that Day = ________

Repeat this process for Thanksgiving (11/26), finding the model temperature for the day and determine the percent error between the model and the actual value, 78, given by the **Weather Underground**. Also, find its corresponding rate of change of temperature.

Model Temperature of Day = ________
Percent Error = ________ Rate of Change of Temperature on that Day = ________

g. Use the model for average high temperature of San Diego to find the maximum high temperature for the year. Also, determine the predicted date for this to occur and find the rate of change on the date. (For the date, enter a number (1-12) for the month and a number (1-31) for the day.)

Maximum high Temperature = ________ on Month = ___ and Day = ___
Rate of Change of Temperature on that Date = ________
Repeat this for the minimum high temperature for the year.
Minimum high Temperature = ________ on Month = ___ and Day = ___
Rate of Change of Temperature on that Date = ________

Also, use this model to predict when the rate of change of the average high temperature is highest and lowest, and what the rate of change is:

Maximum rate of change of the high Temperature occurs on Month = ___ and Day = ___
Rate of Change of Temperature on that Date = ________
Minimum rate of change of the high Temperature occurs on Month = ___ and Day = ___
Rate of Change of Temperature on that Date = ________

h. In your Lab report, create a graph of $T(t)$ and include the data points. Describe how well this graph fits the data. Write a brief paragraph connecting the features of your graph to what is observed with patterns of temperature over the period of a year. Examine your values from Part g for the rate of change of temperature for this model and describe what is occurring on the graph for the highest and lowest values of the rate of change and the times when the rate of change is near zero. Discuss the similarities and differences between the length of daylight and temperature models. Explain which model is better fit by a trigonometric model and why you would expect this.