

1. (1 pt) mathbioLibrary/setABioc2Labs/Lab122.L1.fish.pg

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

The growth of fish has been shown to satisfy a model given by the von Bertalanffy equation:

$$L(t) = L_{\infty}(1 - e^{-bt}),$$

where L_{∞} and b are constants that fit the data. Modeling from before has shown that there is often an allometric model relating the weight and length of different animals. A model relating the weight of a fish as a function of its length is given by

$$W(L) = kL^a,$$

where k and a are constants that fit the data.

a. Below are growth data for the Striped Marlin (*Tetrapturus audax*) [1].

Age (yr)	Length (m)	Age (yr)	Length (m)
1	0.89	6	1.83
2	1.32	7	1.85
3	1.59	8	1.86
4	1.72	9	1.87
5	1.79	10	1.88

Find the least squares best fit of the data to the von Bertalanffy equation above. Give the values of the constants L_{∞} and b and write the model with these constants. Include the value of the least sum of squares error fitting the data.

$$L_{\infty} = \text{_____ m}$$

$$b = \text{_____}$$

$$L(t) = \text{_____ m}$$

$$SSE = \text{_____}$$

Find the L -intercept and the horizontal asymptote for the length of the Striped Marlin.

$$L\text{-intercept} = \text{_____ m}$$

$$\text{Horizontal Asymptote } L = \text{_____ m}$$

Give the model prediction at age 6 and 10 and find the percent error at each of these ages from the actual data given:

$$\text{Length at age 6} = \text{_____ m}$$

$$\text{Percent Error at 6} = \text{_____}$$

$$\text{Length at age 10} = \text{_____ m}$$

$$\text{Percent Error at 10} = \text{_____}$$

b. In your Lab report, create a graph with the data and the von Bertalanffy model for $t \in [0, 15]$. Create a short paragraph that briefly describes the rate of growth of this fish from the graph and what the maximum size of this fish can be. Include how well the model simulates the data.

c. Below are data on the length and weight for the Striped Marlin [2].

Length (m)	Weight (kg)	Length (m)	Weight (kg)
1.02	9	1.39	28
1.1	10	1.48	32
1.17	15	1.54	34
1.25	20	1.7	58
1.28	23	1.79	64

Use Excel's Trendline (Power Law) to find an allometric model of the form above. Give the value of the constants k and a (to at least 5 significant figures) and write the model with these constants. Graph the data and the model.

$$k = \text{_____}$$

$$a = \text{_____}$$

$$W(L) = \text{_____ kg}$$

$$SSE = \text{_____}$$

Give the model prediction for the weight of the Striped Marlin with lengths 1.25 and 1.7 and find the percent error at each of these lengths from the actual data given:

$$\text{Weight at length 1.25} = \text{_____ kg}$$

$$\text{Percent Error at 1.25} = \text{_____}$$

$$\text{Weight at length 1.7} = \text{_____ kg}$$

$$\text{Percent Error at 1.7} = \text{_____}$$

Use information about maximum length of a Striped Marlin to estimate the maximum weight that a Striped Marlin obtains:

$$\text{Maximum weight} = \text{_____ kg}$$

d. In your Lab report, create a graph with the data and the allometric model found above. Create a short paragraph that briefly describes this graph and describe how well the model simulates the data.

e. Create a composite function to give the weight of the Striped Marlin as a function of its age, $W(t)$.

$$W(t) = \text{_____ kg}$$

Find the intercepts and any asymptotes for $W(t)$.

$$W\text{-intercept} = \text{_____ kg}$$

$$\text{Horizontal Asymptote } W = \text{_____ kg}$$

Find the derivative of $W(t)$ using the chain rule. Determine the weight and rate of change in weight at $t = 8$.

$$W'(t) = \text{_____ kg/yr}$$

$$W(8) = \text{_____ kg}$$

$$W'(8) = \text{_____ kg/yr}$$

Also, compute the second derivative, then determine when this second derivative is zero, t_p . From this information, find at what age the Striped Marlin are increasing their weight the most and determine what that weight gain is.

$$\text{Point of Inflection, } t_p = \text{_____ yr}$$

$$W(t_p) = \text{_____ kg}$$

$$W'(t_p) = \text{_____ kg/yr}$$

f. In your Lab report, create a graph the weight of a Striped Marlin as it ages, $W(t)$. Also, create a graph of the derivative, $W'(t)$. Write a short paragraph describing these graphs. Include

a discussion explaining the significance of the point of inflection in the first graph and how it is reflected in the second graph. Summarize your modeling efforts in this lab and briefly discuss the strengths and weaknesses of these models.

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[1] M. G. Hinton. Status of Blue Marlin in the Pacific Ocean. Website accessed 1/04.

[2] J. H. Uchiyama and T. K. Kazama. Updated Weight-on-length relationships for pelagic fishes caught in the central north Pacific Ocean and bottomfishes from the Northwestern Hawaiian Islands, [www.nmfs.hawaii.edu/adminrpts/PIFSC Admin Rep 03-01.pdf](http://www.nmfs.hawaii.edu/adminrpts/PIFSC_Admin_Rep_03-01.pdf), (accessed 1/04)