1. **(1 pt) mathbioLibrary/setABioc2Labs/Lab122_K1_cadium.pg**
Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

Cadium is a toxic heavy metal used in nickel-cadium batteries and cadium telluride solar panels. However, because of its toxicity its use has significantly decreased in other applications. Human exposure to cadium (Cd) comes from two primary sources. It can be ingested, often with leafy vegetables, raw potatoes, and certain meats, where about 0.5-1.0 μg/day are retained. It is much more readily absorbed through the lungs from cigarette smoke, often doubling the intake in the body. The metal concentrates in the kidney tissue. High exposure can cause itai-itai disease and renal failure **(cadium poisoning).** Lower exposure has been linked to the increased risk of cancer **(cadium and smoking).**

a. Cadium is poorly removed from the body and accumulates in the kidney. A differential equation describing the amount of Cd, $C(t)$, in the kidney of a nonsmoker (in mg) is given by:

$$\frac{dC}{dt} = A - kC, \qquad C(0) = 0,$$

where A represents the amount of Cd entering by ingestion of food, k represents the removal rate, and t is in years. Find the solution of this differential equation in terms of A and k.

$C(t) =$ ____________________

b. Below are data for the total Cd in the kidney (in mg) for an average nonsmoker at different ages [1].

Age (yr)	6	14	26	33	43	53
$C(t)$	0.29	0.84	1.22	1.52	1.63	1.75

Find the least squares best fit of the data to the solution of the differential equation above. Give the values of the constants A and k and write the model with these constants. Include the value of the least sum of squares error fitting the data.

$A =$ ______

$k =$ ______

$C(t) =$ ______________ mg

$SSE =$ ______

c. The risk of cancer from cadium is computed by the exposure to this element. The exposure, $E(t)$, is found by the amount of Cd in the tissue times the amount of time that it remains in the tissue. This is readily computed by the integral, which is given by:

$$E(t) = \int_0^t C(s)ds.$$

$E(t) =$ ____________________________

Use this formula and the computed model, $C(t)$, to determine the exposure of the average nonsmoker at ages 30, 50, and 70. Find the exact value of the integral, then use both the Midpoint and Trapezoid Rules with a stepsize of $h = 5$ to approximate each of the integrals.

At age 30, the exact value is

$E(30) =$ ______ mg-yr

The Midpoint Rule with $h = 5$ gives

Midpoint $E(30) =$ ______ mg-yr

The Trapezoid Rule with $h = 5$ gives

Trapezoid $E(30) =$ ______ mg-yr

At age 50, the exact value is

$E(50) =$ ______ mg-yr

The Midpoint Rule with $h = 5$ gives

Midpoint $E(50) =$ ______ mg-yr

The Trapezoid Rule with $h = 5$ gives

Trapezoid $E(50) =$ ______ mg-yr

At age 70, the exact value is

$E(70) =$ ______ mg-yr

The Midpoint Rule with $h = 5$ gives

Midpoint $E(70) =$ ______ mg-yr

The Trapezoid Rule with $h = 5$ gives

Trapezoid $E(70) =$ ______ mg-yr

Find the age at which the average nonsmoker achieves an exposure level of 100 mg-yr.

$E(t_{100}) = 100$ when $t_{100} =$ ______ yr

d. In your Lab report, create a graph with the data and the Cadium model for $t \in [0,70]$. Briefly describe how well the model simulates the data. Create a second graph of the model of exposure to Cd for $t \in [0,70]$for an average nonsmoker. Briefly describe what this graph is saying about the risk of cancer from Cd for a nonsmoker as someone ages.

e. As noted above, lungs absorb cadium much more readily than the gut, so the Cd in cigarettes can easily double the intake of Cd. Because of the carcinogenic properties of Cd, this further increases the cancer risk from smoking. Assume that a smoker begins at age 20. As a simplifying assumption, we will assume that the smoker smokes the same amount of cigarettes annually, and that this increases the Cd intake by a factor of 2.2. For the first 20 years, the amount of Cd entering the body of the smoker is the same as the nonsmoker, following the differential equation in Part a above. For the remainder of the time in this problem, the differential equation describing the amount of Cd, $C_1(t)$, in the kidney of the smoker (in mg) satisfies:

$$\frac{dC_1}{dt} = 2.2A - kC_1, \qquad C_1(20) = C(20),$$

where A and k are the values calculated above. You compute $C(20)$ using your solution from Part a. Find the solution of this initial value problem for $t \geq 20$.

$C_1(t) =$ ____________________

f. Again, the exposure, $E_1(t)$, is found by the amount of Cd in the tissue times the amount of time that it remains in the tissue. The first 20 years are found with the same formula as given in Part c, so $E_1(t) = E(t)$. However, the increased Cd in tobacco results in a new formula for $E_1(t)$ for $t \geq 20$. This is computed by the integral, which is given by:

$$E_1(t) = \int_0^{20} C(s)ds + \int_{20}^{t} C_1(s)ds.$$

$E_1(t) =$ ______________________________ for $t \geq 20$.

Use this formula and the models, $C(t)$ and $C_1(t)$, to determine the exposure of this smoker at ages 30, 50, and 70. Find the exact value of the integral, then use both the Midpoint and Trapezoid Rules with a stepsize of $h = 2$ to approximate all of the integrals.

At age 30, the exact value is
$E_1(30) =$ ________ mg-yr
The Midpoint Rule with $h = 2$ gives
Midpoint $E_1(30) =$ ________ mg-yr
The Trapezoid Rule with $h = 2$ gives
Trapezoid $E_1(30) =$ ________ mg-yr

At age 50, the exact value is
$E_1(50) =$ ________ mg-yr
The Midpoint Rule with $h = 2$ gives
Midpoint $E_1(50) =$ ________ mg-yr
The Trapezoid Rule with $h = 2$ gives
Trapezoid $E_1(50) =$ ________ mg-yr

At age 70, the exact value is
$E_1(70) =$ ________ mg-yr
The Midpoint Rule with $h = 2$ gives
Midpoint $E_1(70) =$ ________ mg-yr
The Trapezoid Rule with $h = 2$ gives
Trapezoid $E_1(70) =$ ________ mg-yr

Find the age at which this smoker achieves an exposure level of 100 mg-yr.
$E_1(ts_{100}) = 100$ when $ts_{100} =$ ________ yr

g. In your Lab report, create a graph with the cadium models for the average nonsmoker, $C(t)$, and particular smoker, $C_1(t)$, for $t \in [0, 70]$. Briefly describe the differences in the amount of Cd between the two models. Create a second graph of the two models, $E(t)$ and $E_1(t)$, for exposure to Cd for $t \in [0, 70]$. Briefly describe what this graph is saying about the relative risk of cancer from Cd for a smoker compared to a nonsmoker as they age. Include in this discussion a comparison of the ages of a smoker versus a nonsmoker when they achieve the exposure level of 100 mg-yr. In this part of the problem, the stepsize, h, was decreased. What happened to the approximations from the Midpoint and Trapezoid Rules relative to the exact integral value? (You might want to mention changes in percent error.)

[1] Lars Friberg, Cadium and the kidney, Environ. Health Persp. (1984), 54, 1-11.