

**1. (1 pt) mathbioLibrary/setABioc2Labs/Lab122\_J4\_blood\_flow.pg**

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

Blood flows through the arteries and veins of the body to carry nutrients and oxygen to tissues and to carry away waste products and carbon dioxide (along with numerous other functions). Fluids stick to surfaces (no slip boundary condition), which results in no fluid flow near any surface. For a small artery with a radius of  $R = 0.061$  cm, blood satisfies Poiseuille's law, which states that the velocity of blood  $r$  cm from the central axis of the artery is given by:

$$v(r) = k(R^2 - r^2),$$

where  $k = 4000$  (per cm-sec) for this specific artery.

a. Find the velocity of the blood at  $r = 0.01$  and  $0.025$  cm. Determine the maximum velocity of blood in this small artery.

$$v(0.01) = \text{_____ cm/sec.}$$

$$v(0.025) = \text{_____ cm/sec.}$$

Maximum velocity,  $v_{max} = \text{_____ cm/sec}$ , At  $r = \text{_____ cm}$ .

b. The rate of flow of blood through this artery can be found by integrating the velocity over the cross sectional area of the blood vessel. In particular, if one divides the artery into small concentric rings with radii  $r$  and  $r + \Delta r$ , then the area of the concentric ring is

$$\pi((r + \Delta r)^2 - r^2) = 2\pi r \Delta r + \pi \Delta r^2,$$

which for small  $\Delta r$  can be approximated by  $2\pi r \Delta r$ . The flow rate is approximated by taking the cross sectional area of each concentric ring times the velocity of the blood flowing through

that concentric ring, then add together all the small concentric rings. In the limit of very small  $\Delta r$ , we obtain the following integral (where the  $\Delta r$  is replaced by  $dr$ ):

$$F = 2\pi k \int_0^R (R^2 - r^2) r dr.$$

With the information above, find the flow rate in  $\text{cm}^3/\text{sec}$  for this small artery.

$$F = \text{_____ cm}^3/\text{sec.}$$

c. The average velocity for the blood in the artery is found by taking the flow rate and dividing by the cross-sectional area. Find the average velocity of blood for this small artery.

$$\text{Cross-sectional Area} = \text{_____ cm}^2.$$

$$\text{Average Velocity} = \text{_____ cm/sec.}$$

d. Suppose that this small artery bifurcates into two smaller arteries each with a radius of  $0.046$  cm. Assuming that the blood flows equally into each of these smaller arteries, then by conservation of blood in these vessels, they each have one half the flow rate found in Part b. Find the value of  $k_s$  for these smaller arteries. Find the velocity of the blood at  $r = 0.01$  cm. What is the maximum velocity of the blood in these smaller arteries?

$$k_s = \text{_____ /cm-sec.}$$

$$v(0.01) = \text{_____ cm/sec.}$$

$$\text{Maximum velocity, } v_{max} = \text{_____ cm/sec.}$$

e. In your Lab Report, create a graph the velocity of blood as a function of the radius  $r$  cm (for  $r \in [0, R]$ ) from the central axis for the original small artery. To this graph add the velocity profile of the smaller arteries from Part d that bifurcate off the original small artery for  $r$  cm (for  $r \in [0, 0.046]$ ). Briefly discuss the significance of these calculations on the physiology of blood flow. What happens to the velocity of the blood flowing as it goes from the aorta to the capillaries?