1. **(1 pt) mathbioLibrary/setABioc2Labs/Lab122_H3_velocity_ball.pg**

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

This problem examines the flight of a baseball. We will ignore the effects of air resistance and try to determine how far we can throw the ball. Assume that we can throw the ball with a velocity of $v_0 = 36$ m/sec.

Find this velocity in miles per hour (mph)

$v_0 =$ _____ mph

We want to determine how far can we throw the ball as we vary the angle α.

(**See Figure.**)

a. We want to follow the trajectory of the ball in its x and y coordinates, assuming the only force acting on the ball is gravity in the y-direction (which is -9.8 m/sec^2). If the initial velocity is v_0 with an angle of α to the ground, then the x-component of the velocity is $v_0 \cos(\alpha)$ and the y-component is $v_0 \sin(\alpha)$. If $v(t)$ and $w(t)$ are the velocities in the x and y directions respectively, then Newton's Law of motion gives the following initial value problem describing the velocity of the ball:

$$\frac{dv}{dt} = 0, v(0) = 36\cos(\alpha)$$

$$\frac{dw}{dt} = -g = -9.8, w(0) = 36\sin(\alpha)$$

Solve for the velocities in the x and y directions. In your answers, use a for α. (Note that this is simply finding the anti-derivatives for the right hand side of each of the differential equations above.)

$v(t) =$ _____ m/sec

$w(t) =$ _____ m/sec

b. By taking one more anti-derivative and using the initial positions $x(0) = 0$ and $y(0) = 0$, you can determine the positions $x(t)$ and $y(t)$ as functions of time. Write the solutions $x(t)$ and $y(t)$ as functions of t and α.

$x(t) =$ _____ m

$y(t) =$ _____ m

c. Consider a ball thrown at an angle $\alpha = 0.27$ (in radians) and where the ball still has an initial velocity, 36 m/sec. Determine the maximum height of the ball.

Maximum height = _____ m

Find how far the ball goes.

Distance = _____ m

How long is the ball in the air?

Time of flight = _____ sec

d. Now consider a ball thrown at an angle $\alpha = 1.29$ (in radians). Determine the maximum height of the ball.

Maximum height = _____ m

Find how far the ball goes.

Distance = _____ m

How long is the ball in the air?

Time of flight = _____ sec

e. For arbitrary α, find when the ball hits the ground by solving $y(t) = 0$ for t, giving the value of t as a function of α (where again you use a for α).

$t_{hit} =$ _____ sec

To find the maximum distance you substitute this value of t into the equation for $x(t)$, then maximize x as a function of α. The distance the ball travels as a function of α is

Distance $x(\alpha) =$ _____ m

Give the angle for the path that maximizes the distance that the ball travels

$\alpha_{max} =$ _____ radians

The maximum distance is

Maximum distance = _____ m

How high does this ball go?

Maximum height = _____ m

How long is this ball in the air?

Time of Flight = _____ sec

f. In your lab report, create a single graph of the trajectories in the x and y plane showing the path of the ball for each of the throws above (with $\alpha = 0.27$, $\alpha = 1.29$, and $\alpha = \alpha_{max}$). Briefly discuss what you observe about the flight of the ball for the various angles, including the length of time in the air, distance traveled, and height the ball achieves.

g. In this part of the problem we want to hit a specific target (say the cutoff man or woman). Find the angles (and there are two of them) that will cause the ball to hit the ground after traveling 69.7 meters. The smaller angle of flight satisfies

$\alpha_1 =$ _____ radians

How high does this ball go?

Maximum height = _____ m

How long is this ball in the air?

Time of Flight = _____ sec

The larger angle of flight satisfies

$\alpha_2 =$ _____ radians

How high does this ball go?

Maximum height = _____ m

How long is this ball in the air?

Time of Flight = _____ sec

h. In your lab report, create a graph of both trajectories from Part g on a single graph, labeling the paths with their respective times of flight. Which of these paths is a baseball fielder using when they are trying to get someone out during a game and why?