**1.** **(1 pt) mathbioLibrary/setABioc2Labs/Lab122_G4_nonlin_cell_growth.pg**
Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

Cells absorb nutrients through their cell membranes, so their growth is proportional to their mass raised to the 2/3 power. This problem examines a group of cells that is transferred into a new medium. Their growth is initially slow as they adapt to the new medium, then it accelerates. However, with time the concentration declines, slow the rate of growth for the cells decline.

a. Suppose that an empirical study shows that the growth rate for the total mass, $M$, (in mg) of a particular species satisfies the following differential equation:

$$\frac{dM}{dt} = \frac{0.8t}{t^2+1}M^{2/3} = f(t,M), \qquad M(0) = 6.2 \text{ mg},$$

where $t$ is in hours. Use Maple's dsolve (or any other technique) to find the solution of this differential equation.
$M(t) =$ ________________________ .

Use this solution to find the length of time for the mass to double in this culture.

Doubling time = _______ hr

b. Earlier we studied discrete dynamical models. Euler's method is a simple extension of these discrete models that we have studied for differential equations. In this part we use Euler's method to numerically approximate the solution to the initial value problem above. Euler's formula is given by:

$$M_{n+1} = M_n + hf(t_n, M_n) = M_n + h\frac{0.8t_n}{t_n^2+1}M_n^{2/3}$$

with $M_0 = 6.2$. Use this Euler's formula with $h = 0.2$ to approximate the solution various times, then compare the approximation to the true solution.

For time t = 1
Actual Solution $M(1) =$ _______ mg
Eulers Solution $M_5 =$ _______ mg
Percent Error = _______
For time t = 2
Actual Solution $M(2) =$ _______ mg
Eulers Solution $M_{10} =$ _______ mg
Percent Error = _______
For time t = 5
Actual Solution $M(5) =$ _______ mg
Eulers Solution $M_{25} =$ _______ mg
Percent Error = _______
For time t = 10
Actual Solution $M(10) =$ _______ mg
Eulers Solution $M_{50} =$ _______ mg
Percent Error = _______

Find the greatest percent error for the entire Euler simulation (which will be negative for this problem) and determine the time in the simulation when this occurs.

Largest percent error (most negative) = _______
Time of this error = _______ hr

c. Repeat the process in Part b. However, Use the Euler's formula with $h = 0.1$ to approximate the solution the same times and compare the approximation to the true solution.

For time t = 1
Eulers Solution $M_{10} =$ _______ mg
Percent Error = _______
For time t = 2
Eulers Solution $M_{20} =$ _______ mg
Percent Error = _______
For time t = 5
Eulers Solution $M_{50} =$ _______ mg
Percent Error = _______
For time t = 10
Eulers Solution $M_{100} =$ _______ mg
Percent Error = _______

Find the greatest percent error for the entire Euler simulation (which will be negative for this problem) and determine the time in this simulation when this occurs.

Largest percent error (most negative) = _______
Time of this error = _______ hr

d. In your Lab Report, create an Excel graph of the actual solution from Part a and the numerical solution from Part b on the domain $t \in [0, 10]$. Adjust the vertical axis of your graph so that the range goes from roughly the minimum of these solutions to the maximum of the solutions on the given domain. Write a brief paragraph on how well the numerical solution represents the actual solution of the differential equation. Briefly describe how the graph fits the biological description given above and note what is happening to the graph near the point where the maximum error occurs in the simulation. Can you determine the relationship between the error in the simulation and the stepsize $h$ used? (Hint: Compare the maximum errors.)