1. (1 pt) mathbioLibrary/setABioc2Labs/Lab122_D1_opt_trough.pg

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

This problem examines two classic problems in optimization that use trigonometric functions. In both problems we use the same amount of material to construct a feeding trough (not including the ends). The feeding troughs are made by bending a 140 cm long strip of sheet metal that is 48 cm wide. The metal sheet is bent so that the 48 cm width creates a trapezoid with the trough maintaining a length of 140 cm.

a. For the first trough the sheet metal is bent into an isosceles trapezoid by turning up strips that are x = 16 cm wide on each side so that they make the same angle, θ, relative to a line perpendicular to the bottom.

(**See Figure A.**)

Write a function of the volume depending on θ. (For your WeBWorK answer, type t for θ.)

$V_A(t) =$ ______ .

Find the volume at $\theta = 0, \pi/4, \pi/2$.

$V_A(0) =$ ______ .

$V_A(\pi/4) =$ ______ .

$V_A(\pi/2) =$ ______ .

b. In your lab report, create a graph the volume as a function of the angle θ for $\theta \in [0, \pi/2]$. Describe the shape of the trough as θ varies through its range of values.

c. Find the derivative of the volume function, $V_A(\theta)$. (Again, for your WeBWorK answer, type t for θ.)

$V_A'(t) =$ ______ .

Find the critical angle, θ_c, and the width across the top that maximizes the volume of this trough. What is the depth and volume of the trough at this optimal solution?

$\theta_c =$ ______ .

Width of Top = ______ .

Depth of Trough A = ______ .

Optimal Volume A = ______ .

d. For the second trough the sheet metal is bent into a right trapezoid with the left side always making a right angle with the bottom and the right side bent such that it makes an angle θ with a line perpendicular to the bottom. Unlike the first trough, where the strip is bent evenly into thirds, you need to determine where the bends are made such that the bottom and the right side are equal and the left side is exactly the height needed to make the top parallel to the bottom.

(**See Figure B.**)

Determine the positions of the bends in the sheet metal as a function of θ. Based on Figure B, find the values of x and h as θ varies. (For your WeBWorK answer, type t for θ.)

$x(t) =$ ______ .

$h(t) =$ ______ .

Write a function of the volume depending on θ. (For your WeBWorK answer, type t for θ.)

$V_B(t) =$ ______ .

Find the volume at $\theta = 0, \pi/4, \pi/2$.

$V_B(0) =$ ______ .

$V_B(\pi/4) =$ ______ .

$V_B(\pi/2) =$ ______ .

e. In your lab report, create a graph the volume as a function of the angle θ for $\theta \in [0, \pi/2]$. Describe the shape of the trough as θ varies through its range of values.

f. Find the derivative of the volume function, $V_B(\theta)$. (Again, for your WeBWorK answer, type t for θ.)

$V_B'(t) =$ ______ .

Find the critical angle, θ_c that maximizes the volume of this trough. Use this critical angle to find the best position x for one bend in the metal sheet and the width across the top. What is the depth and volume of the trough at this optimal solution?

$\theta_c =$ ______ .

$x_{max} =$ ______ .

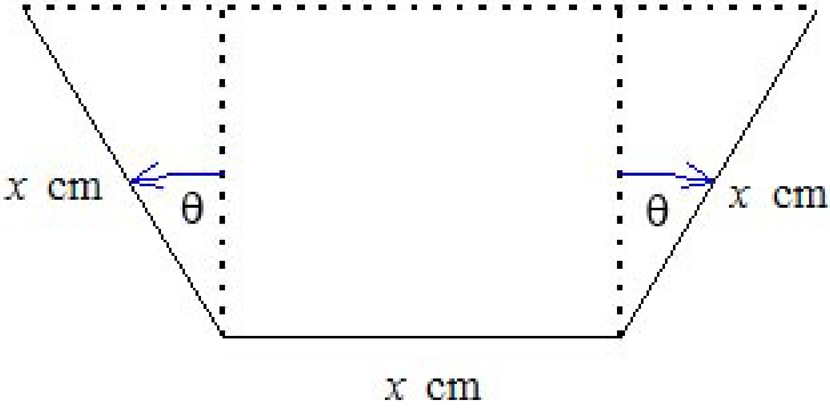
Width of Top = ______ .

Depth of Trough B = ______ .

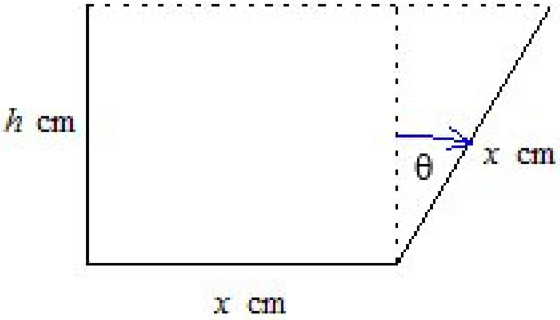
Optimal Volume B = ______ .

Which trough gives the larger volume from these calculations?

Larger Trough (Type either A or B) ___



x cm
θ
θ
x cm
x cm



h cm
θ
x cm
x cm