1. **(1 pt) mathbioLibrary/setABioc2Labs/Lab122_C2_tides.pg**

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on this problem.

This Problem is currently set for tides of 2009, using information about Scripps Pier in San Diego.

The trigonometric functions are often used to generate tide tables. Tides are governed primarily by the forces of gravity from the moon and the sun. However, it is much more complicated than simply just adding these two forces. Many of the tide tables are generated using the sum of 12 or more trigonometric functions based on varying forces due to the elliptical orbits of the Earth and moon and the revolution of the Earth. In this problem, we will use a reduced set of functions to approximate the tides for San Diego in May 2009. There are four dominant forces affecting the tides. The diurnal components (once per day) are denoted K_1, the lunisolar force, and O_1, the main lunar force. The semidiurnal components (twice each day) are given by M_2 the main lunar force, and S_2, the main solar force. The table below gives the period, P_i, in hours for each of these components. This lab uses Excel's Solver to find the amplitude, A_i, in feet, and phase, ϕ_i, in radians needed to predict the tides for May 2009. Because of the semidiurnal forces, there are usually two high tides and two low tides each day.

Name	Period (hr) P_i	Amp (ft)	Phase (rad)
K_1	23.934	A_1	ϕ_1
O_1	25.819	A_2	ϕ_2
M_2	12.421	A_3	ϕ_3
S_2	12	A_4	ϕ_4

The mathematical model for the height, $h(t)$, of the tides is given by the formula:

$$h(t) = A_0 + \sum_{i=1}^{4} A_i \sin\left(\frac{2\pi}{P_i}(t-\phi_i)\right).$$

For uniqueness we choose the parameters for amplitude, $A_i \geq 0$, and principle phase shift, $\phi_i \in [0, P_i)$.

a. The first task in this lab is to complete the table above by finding the least squares best fit of the model to data from actual tide tables for San Diego for May 2009. As we have done in previous lab exercises, we use Excel's solver to find this best fit of $h(t)$ to the data by varying the coefficients, A_i and ϕ_i, in the formula above and minimizing the sum of square errors between the model and the data for high and low tides. To save you considerable time, a spreadsheet of the data and some entries for the parameters of the model are provided.

Tides for 2009 - Scripps Pier, La Jolla

After reading the instructions on the first page, you select the tab at the bottom labeled May 09. The first column contains the time in hours from midnight beginning May 2009. The second column converts the time in hours to (decimal) days from midnight beginning May 2009. The third column gives the predicted tide height in feet. On the right side, the different forces are labeled. **Leave the values fixed for** P_i (labeled p1t,...). For the parameters, A_i and ϕ_i, $i = 1, \ldots, 4$, begin with 1 and 0, respectively. Average the values in Column C (tide height) to obtain your initial estimate for A_0. Find the values of the coefficients A_i and ϕ_i, including A_0, which are determined by Excel's Solver routine. For uniqueness you may need to apply your knowledge about trigonometric functions to shift the phase to obtain the amplitude, $A_i \geq 0$, and principle phase shift, $\phi_i \in [0, P_i)$. Also, give the value of the sum of square errors calculated on the spreadsheet.

$A_1 =$ _____ and $\phi_1 =$ _____
$A_2 =$ _____ and $\phi_2 =$ _____
$A_3 =$ _____ and $\phi_3 =$ _____
$A_4 =$ _____ and $\phi_4 =$ _____
$A_0 =$ _____
SSE = _____

Based on the amplitudes (in absolute value), which force is the strongest? K_1, O_1, M_2,or S_2?

Strongest Force = _____ (Enter K1, O1, M2, or S2.)

b. In your Lab report, create a graph for the period of May 20 - May 27. Include both the model and the data points for the actual high and low tides. (Be careful to select the correct data from the spreadsheet, which starts with $t = 0$ at the beginning of the month, so May 1 is day 0-1. Thus, May 15 would include time (days) in the range 14.0-15.0.) For the horizontal axis use units of days corresponding to the dates. Do NOT forget that the time used in the spreadsheet is in hours, so you will need to convert the hours back to days by when you produce your graph. Because this function oscillates so rapidly, you will need to use about 369 points (which corresponds to simulating the model every 1/2 hour). Discuss how well the model fits the data over this range of the month.

c. From the graph that you generated for your Lab report for the period of May 20 - May 27, you observe different heights for the highest and lowest tides each day. Find the time (in hours after the beginning of the month) and height of the highest and lowest tides during May 20 - May 27 according to the model (absolute maximum and absolute minimum). Note that for some months the model's absolute maximum or minimum may not correspond to the absolute maximum or minimum for the data. For comparison purposes, use the high or low tide (time and height) from the data that matches the time from the model's absolute maximum or minimum. Assuming the values in the table are accurate, then determine the percent error and difference in minutes from the actual timing of the highest or lowest tide (model - actual).

Model Height of highest tide = _____ ft and Percent Error = _____

Model Time _____ hr and Difference from actual = _____ min

Model Height of lowest tide = _____ ft and Percent Error = _____

Model Time _____ hr and Difference from actual = _____ min

The moon phases can be found

Moon Phases.

Which phase of the moon is closest to these highest tides for May 2009?

Moon Phase (New, First Quarter, Full, and Last Quarter) = _____

d. In your Lab report, describe how well the model matches the highest and lowest tides during May 20 - May 27. Does the model actually find the highest and lowest tides according to the data. Discuss the importance of the sun and the moon on tides, and determine which one is more significant based on the amplitudes of the forces that you calculated. Give an explanation for why the highest and lowest tides of the month occur when they do. We noted that most days of the month have two high tides and two low tides. Make a graph for May 6, 2009, showing the model and the data points. Use approximately 100 points to create your graph. Discuss how well the model fits the data for this particular day.

e. For May 6, 2009, determine the heights (ft) and times (in hours after the beginning of the month) of the high high tide, low low tide, low high tide, and high low tide. Find the percent error between the heights of the theoretical and actual tide heights. Also, determine the difference in timing of the theoretical and actual tides (in minutes).

Model Height of high high tide = _____ ft and Percent Error = _____

Model Time _____ hr and Difference from actual = _____ min

Model Height of low low tide = _____ ft and Percent Error = _____

Model Time _____ hr and Difference from actual = _____ min

Model Height of low high tide = _____ ft and Percent Error = _____

Model Time _____ hr and Difference from actual = _____ min

Model Height of high low tide = _____ ft and Percent Error = _____

Model Time _____ hr and Difference from actual = _____ min