

All answers need to be written in a clear, succinct manner. Write a brief paragraph summarizing the answers to the problems with each answer clearly stated in a sentence. Supporting graphs should be provided when asked for, but you should not include printouts of spreadsheets. You can create an appendix to a problem, but that should only include significant material to back up your answers.

1. Work the diabetes problems in WeBWorK, including written work and graphs in your assignment.

2. According to a famous diabetologist, the blood glucose concentration of a nondiabetic who has just absorbed a large amount of glucose will be at or below fasting level in 2 hours or less.

a. The deviation $g(t)$ of a patient's blood glucose concentration from its optimal concentration satisfies:

$$\frac{d^2g}{dt^2} + 2\alpha \frac{dg}{dt} + \alpha^2 g = 0,$$

immediately following absorption of a large amount of glucose, where t is in minutes. Show that this patient is normal according to Ackerman *et al*, if $\alpha > \pi/120$ (min), and that this patient is normal according to the famous diabetologist if

$$g'(0) < -\left(\frac{1}{120} + \alpha\right) g(0).$$

b. Suppose that a patient's blood glucose concentration $G(t)$ satisfies the initial value problem:

$$\frac{d^2G}{dt^2} + 0.05 \frac{dG}{dt} + 0.0004G = 0.03,$$

$$\begin{aligned} G(0) &= 150(\text{mg glucose}/100 \text{ ml blood}) \\ G'(0) &= -\alpha G(0)/(\text{min}); \quad \alpha > 0.02042 \end{aligned}$$

immediately after fully absorbing a large amount of glucose. Is this patient diabetic according to Ackerman *et al*? Explain. Is this patient diabetic according to the diabetologist? Explain.

[1] Ackerman, E., Rosevear, J. W., and McGuckin, W. F. (1964). A mathematical model of the glucose tolerance test, *Phys. Med. Biol.*, **9**, 202-213.

[2] Mahaffy, J. M. and Edelstein-Keshet, L., Modeling cyclic waves of circulating T cells in autoimmune diabetes, *SIAM J. Appl. Math.*, **67**, 915-937 (2007).