1. Gonorrhea ranks high among reportable communicable diseases in the United States. This disease is spread by sexual contact and if untreated can result in blindness, sterility, arthritis, heart failure, and possibly death. Gonorrhea has a very short incubation time (3-7 days) and does not confer immunity to those individuals who have recovered from the disease (SIS disease). It often causes itching and burning for males, particularly during urination, while it is often asymptomatic in females. Thus, males tend to seek treatment more often than females.

a. We let x be the fraction of infective females and y be the fraction of infective males. A mathematical model that describes this disease is given by the following system of differential equations:

$$\begin{aligned} \frac{dx}{dt} &= -a_1 x + b_1(1-x)y, \\ \frac{dy}{dt} &= -a_2 y + b_2(1-y)x, \end{aligned}$$

List the most important assumptions that this model is making about this particular disease. Briefly describe what each of the terms in this model describes about the disease. The constant b_1/a_2 is the infective male contact rate, while b_2/a_1 is the infective female contact rate. Briefly explain what these constants are describing.

b. Find the equilibria of this model and determine the stability of these equilibria. The overall infective contact rate is given by

$$R = \frac{b_1 b_2}{a_1 a_2}.$$

Give the condition on R that implies that this disease remains endemic in the population.

c. It is known that males seek treatment quite readily because of the nature of the disease. Suppose that $a_2 = 0.15$ da^{-1}. Less is known about the other constants. The disease is very transmittable between the sexes, and we'll assume a balanced average number of sexual contacts by each sex. Thus, as a first approximation we take $b_1 = b_2$. Data from a large clinical study show that among a group of 20-24 year olds, there are 517 infected males for every 100,000 in the population and 687 infected females for every 100,000 in the population. Use this information to compute the rate constants a_1, b_1, and b_2. Create a phase-portrait showing all equilibria and nullclines (include at least one typical solution to the model). Give the eigenvalues and eigenvectors associated with each equilibrium.

d. From Part c, how much would we need to lower the rates b_i by education to cause this disease to vanish, according to the model? How much would we need to change treatment (screening) of females (a_1) to cause this disease to vanish? What are the strengths of this model for discussing public policy toward sexually transmitted diseases? What are the weaknesses that you see in the model? (List 2-3 significant weaknesses in the model.) Because AIDS is such a problem in today's society, the sexually active population has changed its behavior since the 1970s. Thus, before the AIDS epidemic, do you believe there were more or fewer cases of gonorrhea? Explain your answer.

e. We noted in class that the well-mixed assumption is particularly weak in this model because of the varying practices of human sexuality. Create a new model that divides the population into two groups of males and females. The first group is a well-mixed very sexually active population (x_1 and y_1), (*e.g.*, prostitutes, military personnel, fraternity members, jocks, etc.), while the second group is well-mixed but has many fewer sexual relations, (x_2 and y_2) (*e.g.*, married people, graduate students, geeks, etc.). Create a model using this structured population. Assume there is a limited interaction between the groups (include this in the model), but most interactions are within the groups. Your model should have terms very similar to the model above, but the constants are different. State how each of your constants in this model would compare to the ones you found in Part c. Briefly discuss the advantages of analyzing a model such as this one over the previous model.

f. (Extra Credit) Find constants in the model in Part e that make reasonable sense and are consistent with the predicted comparable values discussed above (state how you obtain the values you choose). Carefully list the assumptions of your model used to choose your parameters. Finally, the net endemic equilibrium (all infected females x_1 and x_2 and all infected males y_1 and y_2) must agree with the values presented in Part c from the large clinical study.