1. a. The population of Canada was 24,070,000 in 1980, while in 1990 it was 26,620,000. Assuming the population is growing according to the principle of Malthusian growth (with no food or space limitations), find the population as a function of time, and determine its doubling time.

b. For the same years, the populations of Kenya were 16,681,000 and 24,229,000, respectively. Find the population of Kenya as a function of time, assuming it too is growing with Malthusian growth. What is Kenya's doubling time for its population?

c. Use these models to project the populations in the two countries in the year 2000. In what year do the populations of Canada and Kenya become equal?

2. An older woman is quite ill, and her daughter finds that she has been running a temperature of 39°C. Over the night, the woman passes away in her sleep, and the daughter discovers her death at 7 AM. At this time the body is found to be 35°C. Two hours later the body temperature is 33.5°C. The woman's bedroom maintained a temperature of 25°C. If the body satisfies Newton's law of cooling,

$$\frac{dH(t)}{dt} = -k(H(t) - T_e),$$

where T_e is the temperature of the bedroom, t is in hours, H is the temperature in °C, and k is the coefficient of heat transfer to be determined (to **4 significant figures**) for this woman. Determine when the woman died (using normal time, hours and minutes).

3. The data in the following table were obtained by J. M. Cushing using a kitchen thermometer heated in an oven to 150°F and then observed to cool. Use Newton's law of cooling to fit these

t (min)	0	1	2	3	4	5	6
Temp (°F)	150	109	94	85	83	81	80

temperatures to the best least squares model for the data. Include your sum of square errors between the model and the data. Find the percent error between the best fitting model and the temperature recorded at $t = 5$.

4. Anca Segall in the Department of Biology at San Diego State University has done many experiments on the bacterium *Staphylococcus aureus*, a fairly common pathogen that can cause food poisoning. Standard growth cultures of this bacterium satisfy the classical logistic growth pattern discussed in class. Below we present the data from one experiment in her laboratory (by Carl Gunderson), where a normal strain is grown using control conditions and the optical density (OD_{650}) is measured to determine an estimate of the number of bacteria in the culture. Find the best fitting logistic growth model to these data. Include your sum of square errors

t (hours)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
OD_{650}	0.032	0.039	0.069	0.11	0.17	0.229	0.261	0.288	0.309	0.327	0.347

between the model and the data.

5. (Harvesting 1) Suppose that a population of fish, $F(t)$ (in thousands), is given by the following model

$$\frac{dF}{dt} = 0.2\,F\left(1 - \frac{F}{100}\right) - hF,$$

where h is the harvesting term from fishing.

a. Give a modeling description of each term in the equation above. State what conditions make this model reasonable and when is it likely to fail.

b. Assume there is no fishing ($h = 0$). Find all equilibria for this model. Sketch a graph of the right hand side of the model, then draw the phase portrait. Determine the stability of all equilibria.

c. Let $h = 0.05$, then find all equilibria for this model. Sketch a graph of the right hand side of the model, then draw the phase portrait. Determine the stability of all equilibria.

d. What level of fishing (value of h) results in the fish going extinct? What type of bifurcation occurs at this value of h?

6. (Harvesting 2) Suppose that a population of fish, $F(t)$ (in thousands), is given by the following model

$$\frac{dF}{dt} = 0.2\,F\left(1 - \frac{F}{100}\right) - h,$$

where h is the harvesting term from fishing.

a. Give a modeling description of each term in the equation above. State what conditions make this model reasonable and when is it likely to fail.

b. Let $h = 1.8$, then find all equilibria for this model. Sketch a graph of the right hand side of the model, then draw the phase portrait. Determine the stability of all equilibria.

c. What level of fishing (value of h) results in the fish going extinct? What type of bifurcation occurs at this value of h?

7. (The Allee effect) For higher organisms, the growth rate is more complicated than the logistic growth model. For example, reproduction could be reduced if it becomes too difficult to find a mate (which is one problem apparently facing the Giant Panda). An alternate higher order model to the logistic growth model is one modeling the Allee effect. A differential equation for this model is given by

$$\frac{dP}{dt} = P\left(r - a(P - b)^2\right),$$

where $b > \sqrt{r/a}$.

a. Find all equilibria and determine the stability of the equilibria. Draw a phase portrait of this model.

b. Compare this model to the logistic growth model. Describe the similarities and differences between these models. Write a brief paragraph discussing how the equation above would relate to some animal population, *i.e.*, give a brief ecological interpretation of the model.