1. Consider the data on the yeast *Schizosaccharomyces kephir* that are given in the lecture notes. This problem has the reader repeat most of the analyses of the lecture notes that was done on the yeast *Saccharomyces cerevisiae* using the discrete Malthusian growth model.

a. From the data, we see that the volume of *S. kephir* was 1.27 at 9 hours and 4.56 at 45.5 hours. Use these data points to find a Malthusian growth model for the growth of this yeast culture.

b. Use the first 6 data points (at $t = 9, 10, 23, 25.5, 42, 45.5$) to find the best fit of an exponential function to the data. Find the best straight line fit through the logarithm of the volume data, then give the best discrete Malthusian growth model. (You can use Excel, MatLab, or any other computational method you want, but be sure to say how you derived your model.) Give the sum of squares error between this model (not logarithm of the model) and the data.

c. Use the first 6 data points (at $t = 9, 10, 23, 25.5, 42, 45.5$) to find the best nonlinear least squares fit of a discrete Malthusian growth model to the data. Unlike the previous case, you do not transform the data with logarithms to make the fit easier. (You can use Excel, MatLab, or any other computational method you want, but be sure to say how you derived your model.) Give the sum of squares error between this model and the data.

d. Graph all 3 of the models above and give a brief discussion of how well they model the growing culture of yeast.

2. a. Consider a population of bacteria that doubles every 25 minutes (which is not uncommon for cultures of *Escherichia coli*). If there are initially 1000 bacteria, then write a Malthusian growth model for this culture.

b. Suppose a mutant individual is introduced at the beginning of an experiment with the mutant strain doubling every 24 minutes. If there are 1000 of the original bacteria and one of the mutant strain, then determine how long until 50% of the culture consists of the mutant strain.

3. Michael Crichton in the *Andromeda Strain* (1969) states that "A single cell of the bacterium *E. coli* would, under ideal circumstances, divide every twenty minutes... [I]t can be shown that in a single day, one cell of *E. coli* could produce a super-colony equal in size and weight to the entire planet Earth." A single *E. coli* has a volume of about 1.7 μm^3. The diameter of the Earth is 12,756 km, so assuming it is a perfect sphere, determine how long it takes for an ideally growing colony (Malthusian growth) of *E. coli* (doubling every 20 min) to equal the volume of the Earth.

4. a. Consider the data on the yeast *Schizosaccharomyces kephir* that are given in the lecture notes. This problem has the reader repeat most of the analyses of the lecture notes that was done on the yeast *Saccharomyces cerevisiae* using the discrete logistic growth model. In Problem 1, you obtained the best growth rate r for the initial exponential growth phase of this yeast population and an estimate of the initial population, P_0. (Use the nonlinear least squares best fit.) Average the last two data points of the *S. kephir* experiments to obtain an estimate

of the carrying capacity M. (Write all of these initial values in your HW.) Use either Excel or MatLab (or any other technique that you can document) to find the best nonlinear least squares fit of the discrete logistic growth model to the data for *S. kephir*, varying the parameters, r, P_0, and M. Write both the best discrete model equation and the best solution to the discrete model in your HW, using the best fitting parameters that you find. Give the sum of squares error between this model and the data.

b. Graph the logistic growth model with your initial estimate and the best fitting parameters along with the data, then give a brief discussion of how well the logistic growth model follows the data for the growing culture of yeast.

5. Using data from the U. S. census bureau, the table below presents the population (in millions) for France. This lab has you repeat for this country the modeling effort that we performed in class for the U. S.

Year	Population
1950	41.83
1960	45.67
1970	50.79
1980	53.87
1990	56.74
2000	59.38

a. Find the growth rate for each decade with the data above by dividing the population from one decade by the population of the previous decade and subtracting 1 from this ratio. Associate each growth rate with the earlier of the two census dates. Determine the average (mean) growth rate, r, from the data above. Associate t with the earlier of the dates in the growth ratio, and use Excel's Tendline to find the best straight line

$$k(t) = a + bt$$

through the growth data. Graph the constant function r, $k(t)$, and the data as a function of t over the period of the census data. It is very important that you click on the Trendline equation and reformat the coefficient b so that it has more significant figures (obtain 4 significant figures for a and b).

b. The Discrete Malthusian growth model is given by

$$P_{n+1} = (1 + r)P_n.$$

where r is computed in Part a. and P_0 is the population in 1950. Write the general solution to this model, where n is in decades. Use the model to predict the population in 2020 and 2050.

c. The revised growth model is given by

$$P_{n+1} = (1 + k(t_n))P_n.$$

where $k(t_n)$ is computed in Part a. and P_0 is again the population in 1950. Simulate this nonautonomous discrete dynamical model from 1950 to 2050. (Note that $t_n = 1950 + 10n$.) Use the model to predict the population in 2020 and 2050.

d. Create a table listing the date, the population data, the predicted values from the Malthusian growth model, the nonautonomous dynamical model, and the percent error between the actual population and each of the predicted populations from the models from 1950 to 2000. What is the maximum error for each model over this time interval? Use Excel to graph the data and the solutions to the each of the models above for the period from 1950 to 2050. Briefly discuss how well these models predict the population over this period. List some strengths and weaknesses of each of the models and how you might obtain a better means of predicting the population.

e. The growth rate of the nonautonomous dynamical model goes to zero during this century for France. At this time, this model predicts that the population will reach its maximum and start declining. Use the growth rate $k(t)$ to find when this model predicts a maximum population, then estimate what that maximum population will be.

f. Use the data above to find the best discrete logistic growth model fit for the population of France. Add the graph of this model to your previous graph of the Malthusian growth models for the time period 1950 to 2050. What does this model predict for the population of France in 2050? From the sum of square errors, which model matches the data best? Find all equilibria of this model and discuss the stability of these equilibria (include the values of the derivatives at the equilibria). What does this model predict will happen over a long period of time for France's population?

6. A. C. Crombie studied *Oryzaephilus surinamensis*, the saw-tooth grain beetle, with an almost constant nutrient supply (maintained 10 g of cracked wheat weekly). These conditions match the assumptions of the discrete logistic model. The data below show the adult population of *Oryzaephilus* from Crombie's study (with some minor modifications to fill in uncollected data and an initial shift of one week).

Week	Adults	Week	Adults
0	4	16	405
2	4	18	471
4	25	20	420
6	63	22	430
8	147	24	420
10	285	26	475
12	345	28	435
14	361	30	480

The discrete logistic growth model for the adult population P_n can be written

$$P_{n+1} = f(P_n) = rP_n - mP_n^2,$$

where the constants r and m must be determined from the data.

a. Plot P_{n+1} vs. P_n, which you can do by entering the adult population data from times 2–30 for P_{n+1} and times 0–28 for P_n. (Be sure that P_n is on the horizontal axis.) To find the appropriate constants use Excel's Trendline with its polynomial fit of order 2 and with the intercept set to 0 (under options). In your lab, write the equation of the model which fits the data best. Graph both $f(P)$ and the data.

b. Find the equilibria for this model. Write the derivative of the updating function. Discuss the behavior of the model near its equilibria. (Note that if P_e is an equilibrium point, then you can determine the behavior of that equilibrium by evaluating the derivative of the updating function $f(P_n)$ at P_e. (For more reference check Chapter 8 of the text book [5]). Simulate the model and show this simulation compared to the data from the table above (adult population vs. time). Discuss how well your simulation matches the data in the table. What do you predict will happen to the adult saw-tooth grain beetle population for large times (assuming experimental conditions continue)?

c. Another common population model is Ricker's, which is given by

$$P_{n+1} = R(P_n) = aP_ne^{-bP_n},$$

where a and b are constants to be determined. Use Excel's solver to find the least squares best fit of the Ricker's updating function to the given data by varying a and b. As initial guesses take $a = 2.5$ and $b = 0.002$. Once again plot P_{n+1} vs. P_n, using this updating function and show how it compares to the data (much as you did in Part a.

d. Find the equilibria for Ricker's model. Write the derivative of the updating function, then discuss the behavior of these equilibria using this derivative. (Give the value of the derivative at the equilibria.) Simulate the discrete dynamical system using Ricker's model. Show the graphs of the logistic and Ricker's models with the data. Compare these simulations with the data. Discuss the similarities and differences that you observe between models and how well they work for this experimental situation.

From the text, work the following problems:

Exercises 2.4.2-5, (p. 48)