

HW 8 9.4 1, 3, 5, 7, 10, 11, 13, 14

$$\begin{aligned} \text{7. } \left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & -4 & -2 & 2 & 6 \\ 4 & 1 & -3 & 3 & -1 \end{array} \right] & \xrightarrow{\substack{R_2 - \frac{1}{2}R_1 \\ R_3 - \frac{1}{2}R_1 \\ R_4 - 2R_1}} \left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & -3/2 & -1/2 & 1/2 & 2 \\ 0 & -5/2 & -3/2 & 3/2 & 7 \\ 0 & -1 & -1 & 1 & 3 \end{array} \right] & \xrightarrow{\substack{-2R_2 \\ -2R_3}} \left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & 3 & 1 & -1 & -4 \\ 0 & 9 & 3 & -3 & -14 \\ 0 & -1 & -1 & 1 & 3 \end{array} \right] \end{aligned}$$

$$\begin{aligned} & \xrightarrow{\substack{R_3 - 3R_2 \\ R_4 + \frac{1}{2}R_2}} \left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & 3 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & -2/3 & 2/3 & 5/3 \end{array} \right] & \xrightarrow{\substack{R_4 \leftrightarrow R_3 \\ 3R_4}} \left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & 3 & 1 & -1 & -4 \\ 0 & 0 & -2 & 2 & 5 \\ 0 & 0 & 0 & 0 & -2 \end{array} \right] \end{aligned}$$

Rank(A) = 3, Rank(A|b) = 4
No solution.

$$\begin{aligned} \text{13. } \det \begin{vmatrix} x & 1 & 0 & 1 \\ 1 & x & 1 & 0 \\ 0 & 1 & x & 1 \\ 1 & 0 & 1 & x \end{vmatrix} &= x \begin{vmatrix} x & 1 & 0 \\ 0 & 1 & x \end{vmatrix} - \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & x \end{vmatrix} - \begin{vmatrix} 1 & x & 1 \\ 0 & 1 & x \end{vmatrix} \\ &= x(x^2 - 1) - (x - 1) - (x^2 - 1) = x^3 - 4x^2 = 0 \therefore x = 0, 0, \pm 2. \\ & \text{Nontrivial soln. when } x = 0, \pm 2 \end{aligned}$$

9.5 3, 4, 5, 6, 7, 9, 12, 13, 16

3. $u = a_3x^3 + a_2x^2 + a_1x + a_0$, $v = b_3x^3 + b_2x^2 + b_1x + b_0$, $w = c_3x^3 + c_2x^2 + c_1x + c_0$
 $u+v = (a_3+b_3)x^3 + (a_2+b_2)x^2 + (a_1+b_1)x + (a_0+b_0) \in V$ closed under addition
 $u+v = v+u$, so commutative, $(u+v)+w = u+(v+w)$, so associative
 Take $a_3 = a_2 = a_1 = a_0 = 0$, then $u \equiv 0$ is the unique zero vector
 If $b_3 = -a_3, b_2 = -a_2, b_1 = -a_1, b_0 = -a_0$, then $u+v = 0$ or $v = -u$.
 $\alpha u = \alpha a_3x^3 + \alpha a_2x^2 + \alpha a_1x + \alpha a_0 \in V$, closed under scalar mult.
 $\alpha(\beta u) = (\alpha\beta)u$ (associative), $\alpha(u+v) = \alpha u + \alpha v$ and $(\alpha+\beta)u = \alpha u + \beta u$, distributive
 $1 \cdot u = u$ (scalar identity) V has 4-dimensions

$$\text{12. } \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 3 \end{array} \right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 3 \end{array} \right] \xrightarrow{\substack{R_2 + R_3 \\ -R_2 \\ R_1 + R_2}} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 4 \end{array} \right] \xrightarrow{R_1 - R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 4 \end{array} \right]$$

Thus $[1, 2, 3] = -2[1, 1, 0] - [1, 0, 1] + 4[1, 1, 1]$