1. a. The least sum of square error satisfies:

\[ \sum_{i=1}^{N} (E_d(t_i) - E(t_i))^2 = 12,086.41. \]

The values of the best fitting parameters are \( a_0 = 217.75, a_1 = 70.945, \omega = 0.19598, \) and \( \delta_1 = 18.287, \) giving the formula:

\[ E(t) = 217.75 + 70.945 \cos(0.19598(t - 18.287)). \]

The period is \( \frac{2\pi}{\omega} = 32.06 \) days. From the formula above, \( E(0) = 153.63, E(7) = 175.31, E(14) = 265.09, \) and \( E(21) = 278.90. \)

b. By adding an additional trigonometric function, the least sum of square error satisfies:

\[ \sum_{i=1}^{N} (E_d(t_i) - E(t_i))^2 = 4,500.194. \]

The values of the best fitting parameters are \( a_0 = 223.705, a_1 = 70.666, a_2 = 37.121, \omega = 0.22723, \delta_1 = 17.802, \) and \( \delta_2 = -2.5986, \) giving the formula:

\[ E(t) = 223.705 + 70.666 \cos(0.22723(t - 17.802)) + 37.121 \cos(0.45446(t + 2.5986)). \]

The period is \( \frac{2\pi}{\omega} = 27.65 \) days. From the formula above, \( E(0) = 194.08, E(7) = 156.33, E(14) = 280.94, \) and \( E(21) = 266.58. \) Note that the sum of square error is a factor of 3 better and the period is closer to the expected 28-days. Below is a graph showing data and the two approximating functions.
2. a. With Euler’s method and $h = 0.2$, the approximate solution to

$$\frac{dP}{dt} = (0.12 - 0.05t)P, \quad P(0) = 200$$

is $P(1) = 220.81$, $P(2) = 233.00$, $P(3) = 232.07$, and $P(5) = 199.48$.

b. With a stepsize of $h = 0.1$, the approximate the population at years $t = 1, 2, 3, \text{ and } 5$ is $P(1) = 220.37$, $P(2) = 231.07$, $P(3) = 230.49$, and $P(5) = 197.26$.

c. The exact solution to this initial value problem is

$$P(t) = 200e^{0.12t-0.025t^2}.$$ The population at years $t = 1, 2, 3, \text{ and } 5$ is $P(1) = 219.93$, $P(2) = 230.05$, $P(3) = 228.91$, and $P(5) = 195.06$. This population reaches a maximum at $t = 2.4$ years with a population of $P(2.4) = 230.98$ thousand.

d. The table below summarizes the values of the true solution, Euler’s approximate solution with $h = 0.2$, the percent error between these, Euler’s approximate solution with $h = 0.1$, and the percent error of this with the true solution.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$P(t)$</th>
<th>Euler $h = 0.2$</th>
<th>% error</th>
<th>Euler $h = 0.1$</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>219.93</td>
<td>220.81</td>
<td>0.4001</td>
<td>220.37</td>
<td>0.2001</td>
</tr>
<tr>
<td>2</td>
<td>230.05</td>
<td>233.00</td>
<td>1.282</td>
<td>231.07</td>
<td>0.4434</td>
</tr>
<tr>
<td>3</td>
<td>228.91</td>
<td>232.07</td>
<td>1.380</td>
<td>230.49</td>
<td>0.6902</td>
</tr>
<tr>
<td>5</td>
<td>195.06</td>
<td>199.48</td>
<td>2.266</td>
<td>197.26</td>
<td>1.128</td>
</tr>
</tbody>
</table>

Notice that the error from Euler’s method decreases by about one half when the stepsize is decreased by one half.