1. a. For the second integral, we let $u = x^2$, so $du = 2x \, dx$, then the integral can be written:

$$6 \int x^2 \, dx - 2 \int \cos(u) \, du = 6 \left( \frac{2}{3} \right) x^3 - 2 \sin(u) + C = 4 x^3 - 2 \sin(x^2) + C.$$ 

b. For the first integral, we let $u = e^{2x} + 4$, so $du = 2e^{2x} \, dx$, the integral can be written:

$$2 \int \frac{du}{u} - 12 \int \sin(6x) \, dx = 2 \ln(u) + 2 \cos(6x) + C = 2 \ln(e^{2x} + 4) + 2 \cos(6x) + C.$$ 

2. a. Both antiderivatives are standard for this problem

$$\int_{0}^{2\pi} (\cos(t/4) + t) \, dt = \left( 4 \sin(t/4) + \frac{t^2}{2} \right)_{t=0}^{t=2\pi} = 4 \sin(\pi/2) + \frac{4\pi^2}{2} - 4 \sin(0) - 0 = 4 + 2\pi^2$$

b. The first integral uses the substitution $u = x^2 + 1$, so $du = 2x \, dx$ with endpoints $u = 2$ when $x = 1$ and $u = 5$ when $x = 2$, while the second antiderivative is the natural logarithm.

$$\int_{1}^{2} \left( \frac{6x}{x^2 + 1} + \frac{2}{x} \right) \, dx = 3 \int_{2}^{5} \frac{du}{u} + 2 \ln|x|_{x=1}^{x=5}$$

$$= 3 \ln|u|_{u=2}^{5} + 2 \ln(2) - 2 \ln(1)$$

$$= 3 \ln(5) - 3 \ln(2) + 2 \ln(2) = 3 \ln(5) - \ln(2) = \ln(62.5) \approx 4.135$$

c. Let $u = 25 - x^2$, so $du = -2x \, dx$ with endpoints $u = 25$ when $x = 0$ and $u = 0$ when $x = 5$.

$$\int_{0}^{5} x\sqrt{25 - x^2} \, dx = -\frac{1}{2} \int_{25}^{0} u^{\frac{3}{2}} du$$

$$= -\frac{1}{2} \left( \frac{u^{3}}{\frac{3}{2}} \right)_{u=25}^{0}$$

$$= -\frac{1}{3} \left( 0^{\frac{3}{2}} - 25^{\frac{3}{2}} \right) = \frac{125}{3}$$

d. For the first integral, let $u = \ln(x)$, so $du = dx/x$ with endpoints $u = 0$ when $x = 1$ and $u = \ln(4)$ when $x = 4$. The second integral is a power rule.

$$\int_{1}^{4} \left( \frac{\ln(x)}{x} + \frac{3}{\sqrt{x}} \right) \, dx = \int_{0}^{\ln(4)} u \, du + 3 \int_{1}^{4} x^{-\frac{1}{2}} \, dx$$

$$= \frac{u^2}{2} \bigg|_{u=0}^{\ln(4)} + 6 x^{\frac{1}{2}} \bigg|_{1}^{4}$$

$$= \frac{1}{2} (\ln(4))^2 + 6 (2 - 1) = 6 + 2 (\ln(2))^2$$
e. For the second integral, let \( u = \ln(x + 1) \), so \( du = \frac{dx}{x+1} \) with endpoints \( u = 0 \) when \( x = 0 \) and \( u = \ln(3) \) when \( x = 2 \).

\[
\int_{0}^{2} \left(4x - \frac{\ln(x + 1)}{x+1}\right) \, dx = 2x^2 \bigg|_{x=0}^{x=2} - \int_{0}^{\ln(3)} u \, du \\
= 8 - \frac{u^2}{2} \bigg|_{u=0}^{u=\ln(3)} \\
= 8 - \frac{1}{2}(\ln(3))^2
\]

3. a. We apply the quotient rule and obtain

\[
f'(t) = \frac{(e^{10t} + e^{-10t})(10e^{10t} + 10e^{-10t}) - (e^{10t} - e^{-10t})(10e^{10t} - 10e^{-10t})}{(e^{10t} + e^{-10t})^2}
\]

\[
= \frac{40}{(e^{10t} + e^{-10t})^2}
\]

This is clearly positive, so \( f(t) \) is always increasing. It is easy to see that the numerator is always smaller than the denominator, so this function must be between \(-1\) and \(1\).

b. To solve the integral, we make the substitution \( u = e^{10t} + e^{-10t} \), so \( du = 10(e^{10t} - e^{-10t}) \, dt \) with endpoints \( u = 2 \) for \( t = 0 \) and \( u = e^{10} + e^{-10} \) for \( t = 1 \). Thus,

\[
\int_{0}^{1} \frac{e^{10t} - e^{-10t}}{e^{10t} + e^{-10t}} \, dt = 0.1 \int_{2}^{e^{10} + e^{-10}} \frac{du}{u} \\
= 0.1 \ln \left| u \right|_{u=2}^{u=e^{10}+e^{-10}} = 0.1(\ln(e^{10} + e^{-10}) - \ln(2)) \\
= 0.1 \ln \left( \frac{e^{10} + e^{-10}}{2} \right) = 0.93
\]

4. a. From the equation of the line, \( y = 3 - x \), the \( x \) and \( y \)-intercepts are easily seen to be \((3, 0)\) and \((0, 3)\), respectively. Also, the slope of the line is \( m = -1 \). The equation for the parabola is \( y = 6 + x - x^2 = -(x + 2)(x - 3) \). From this it is easy to see that the \( x \)-intercepts are \((-2, 0)\) and \((3, 0)\), while the \( y \)-intercept is \((0, 6)\). The vertex of the parabola has its \( x \)-coordinate halfway between the \( x \)-intercepts, thus the vertex is \((\frac{1}{2}, 6)\).

Setting the two equations together, \( 3 - x = 6 + x - x^2 \) or \( x^2 - 2x - 3 = (x + 1)(x - 3) = 0 \). Thus, the points of intersection are \((-1, 4)\) and \((3, 0)\). The graph can be seen below.

b. The area between the two curves is given by

\[
\int_{-1}^{3} (6 + x - x^2 - (3 - x)) \, dx = \int_{-1}^{3} 3 + 2x - x^2 \, dx \\
= \left( 3x + x^2 - \frac{x^3}{3} \right) \bigg|_{x=-1}^{x=3} \\
= 9 + 9 + 3 - 1 - \frac{1}{3} = \frac{32}{3}
\]
5. a. The period of this function is $2\pi$. This is a typical cosine function with an amplitude of 2 and with the graph shifted up by 2 units. The graph can be seen below.

b. The area under the curve is given by

$$\int_0^{2\pi} 2\cos(t) + 2\,dt = (2\sin(t) + 2t)|_{t=0}^{2\pi}$$

$$= 4\pi$$
6. a. The derivative is $A'(t) = -0.2e^{-0.02t} + e^{-0.1t}$. The maximum occurs when $A'(t) = 0$ or $0.2e^{-0.02t} = e^{-0.1t}$. Thus, $e^{0.08t} = 5$ or $t = 12.5\ln(5) \simeq 20.12$ days. The maximum is $A(12.5\ln(5)) \simeq 10 (e^{-0.4024} - e^{-2.012}) = 5.350$ µg. The only intercept is $(0,0)$, and there is a horizontal asymptote $A = 0$. The graph is shown below.

b. The total exposure over 60 days is the following integral:

$$10 \int_0^{60} (e^{-0.02t} - e^{-0.1t}) \, dt = 10 \left. \left( -50e^{-0.02t} + 10e^{-0.1t} \right) \right|_{t=0}^{60} = 10 \left( -50e^{-1.2} + 10e^{-6} + 50 - 10 \right) = 400 + 100e^{-6} - 500e^{-1.2} \simeq 249.65.$$

7. a. This is a linear differential equation, which can be written

$$\frac{dy}{dt} = -0.2(y - 25).$$

With the substitution $z(t) = y(t) - 25$, we have

$$\frac{dz}{dt} = -0.2z, \quad z(0) = y(0) - 25 = -15.$$

Thus, $z(t) = -15e^{-0.2t}$. It follows that

$$y(t) = 25 - 15e^{-0.2t}.$$

b. This is a separable differential equation. It can be written

$$\int \frac{dy}{y} = \int \frac{4t}{t^2 + 4} \, dt.$$
The integral on the right hand side is solved using the substitution \( u = t^2 + 4 \), so \( du = 2t \, dt \).

Integrating we have

\[
\ln |y| = 2 \ln(t^2 + 4) + C = \ln(t^2 + 4)^2 + C.
\]

Exponentiating both sides, we find

\[
|y(t)| = e^{C(t^2 + 4)^2}.
\]

With the initial condition, \( y(0) = 64 = e^{C(4^2)} \), so \( e^C = 4 \). Hence, the solution is

\[
y(t) = 4(t^2 + 4)^2.
\]

\( c \). This is a time-varying differential equation, so we integrate giving

\[
y(t) = \int (8 - 3 \sin(3t)) \, dt = 8t + \cos(3t) + C.
\]

With the initial condition \( y(0) = 5 = 1 + C \), \( C = 4 \). It follows that

\[
y(t) = 8t + \cos(3t) + 4.
\]

8. a. The average is \( A = \frac{4 + 1.5}{\frac{1}{2}} = 2.75 \). The amplitude of the function is \( B = 4 - 2.75 = 1.25 \). The period is 28 days, so the frequency is \( \omega = \frac{\pi}{14} \simeq 0.2244 \). Since cosine has a maximum at 0, it follows that the maximum of the approximating function occurs when the argument is 0 or \( \omega(9 - \phi) = 0 \) or \( \phi = 9 \). Thus, the best function to approximate the concentration of FSH is given by

\[
F(t) = 2.75 + 1.25 \cos \left( \frac{\pi}{14}(t - 9) \right).
\]

With this function the approximate concentration of FSH at ovulation is \( F(14) = 3.292 \) relative units. The graph of this function is shown below.
b. The derivative is $F'(t) = -\frac{1.25\pi}{14} \sin\left(\frac{\pi}{14}(t-9)\right) \approx -0.2805 \sin(0.2244(t-9))$. At ovulation, the concentration of FSH is decreasing at a rate of $F'(14) = -0.2527$ relative units/day.

c. The average concentration of FSH from the beginning of menstruation ($t = 0$) to ovulation ($t = 14$) is given by:

$$\frac{1}{14} \int_{0}^{14} F(t) \, dt = \frac{1}{14} \int_{0}^{14} \left(2.75 + 1.25 \cos\left(\frac{\pi}{14}(t-9)\right)\right) \, dt$$

$$= \frac{1}{14} \left(2.75t|_{0}^{14} + \frac{1.25(14)}{\pi} \left(\sin\left(\frac{\pi}{14}(t-9)\right)\right)|_{0}^{14}\right)$$

$$= 2.75 + 0.3979(\sin(1.1220) - \sin(-2.0196)) = 3.467$$

9. a. The rate of change of amount $a(t)$ of pollutant equals the amount flowing in minus the amount flowing out. So

$$\frac{da}{dt} = f_1 Q - f_2 c = 8(450) - \frac{400a}{500,000}$$

Dividing both sides by the volume of the lake gives the concentration equation:

$$\frac{dc}{dt} = 0.0072 - 0.0008c, \quad c(0) = 0.$$ 

This can be written $\frac{dc}{dt} = -0.0008(c - 9)$, so a substitution of $z(t) = c(t) - 9$ is made, giving the initial value problem

$$\frac{dz}{dt} = -0.0008z, \quad z(0) = c(0) - 9 = -9,$$

which has the solution $z(t) = -9e^{-0.0008t}$. It follows that

$$c(t) = 9 - 9e^{-0.0008t}.$$  

b. For $c(t) = 2 = 9 - 9e^{-0.0008t}$, $e^{0.0008t} = \frac{9}{7}$ or $t = 1250 \ln \left(\frac{9}{7}\right) = 314.14$ days to reach a concentration of $2 \, \mu g/m^3$. The limiting concentration is $9 \, \mu g/m^3$. A graph of the concentration is shown below.
c. The modified equation becomes \( \frac{dc}{dt} = 0.0072 e^{-0.002 t} - 0.0008 c \), so Euler’s formula with \( h = 0.5 \) is given by

\[
c_{n+1} = c_n + 0.5 \left( 0.0072 e^{-0.002 t_n} - 0.0008 c_n \right), \quad c_0 = 0.
\]

<table>
<thead>
<tr>
<th>( t_n )</th>
<th>( c_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_0 = 0 )</td>
<td>( c_0 = 0 )</td>
</tr>
<tr>
<td>( t_1 = 0.5 )</td>
<td>( c_1 = c_0 + 0.5 \left( 0.0072 e^{-0.002 t_0} - 0.0008 c_0 \right) = 0.0036 )</td>
</tr>
<tr>
<td>( t_2 = 1 )</td>
<td>( c_2 = 0.0036 + 0.5 \left( 0.0072 e^{-0.001} - 0.0008(0.0036) \right) = 0.007195 )</td>
</tr>
</tbody>
</table>

It follows that the Euler’s approximation for \( c(1) = 0.007195 \) \( \mu g/m^3 \).

10. According Newton’s Law of cooling, the differential equation for this elderly patient is

\[
\frac{dH}{dt} = -k(H - 72), \quad H(0) = 96 \quad \text{and} \quad H(2) = 94.
\]

Let \( z(t) = H(t) - 72 \), then \( \frac{dz}{dt} = -kz \) with \( z(0) = 96 - 72 = 24 \). Thus, \( z(t) = 24 e^{-kt} \), so \( H(t) = 72 + 24 e^{-kt} \). Since \( H(2) = 94 = 72 + 24 e^{-2k} \), we have \( e^{2k} = \frac{24}{22} \) or \( k = 0.5 \ln \left( \frac{24}{22} \right) \approx 0.043506 \). The solution is

\[
H(t) = 72 + 24 e^{-0.043506 t}.
\]

To find the time of death, we solve \( 102 = 72 + 24 e^{-0.043506 t} \) or \( e^{0.043506 t} = \frac{24}{22} \). The time of death is

\[
t = \frac{\ln \left( \frac{24}{22} \right)}{0.043506} \approx -5.129 \text{ hr},
\]

which is approximately 1:52 AM.

11. a. We divide \( x \in [0, 4] \) into \( n = 4 \) subintervals, then \( x_1 = 0.5,\; x_2 = 1.5,\; x_3 = 2.5,\) and \( x_4 = 3.5 \) with \( \Delta x = 1 \). The midpoint rule gives

\[
\int_0^4 x \sqrt{x^2 + 9} \, dx \approx (0.5 \sqrt{9.25} + 1.5 \sqrt{11.25} + 2.5 \sqrt{15.25} + 3.5 \sqrt{21.25}) \cdot 1 = 32.449.
\]

b. The trapezoid rule gives

\[
\int_0^4 x \sqrt{x^2 + 9} \, dx \approx \left( \frac{1}{2} (0) + \sqrt{10} + 2\sqrt{13} + 3\sqrt{18} + 2\sqrt{25} \right) \cdot 1 = 33.101.
\]

c. To evaluate this integral exactly, we substitute \( u = x^2 + 9 \), so \( du = 2x \, dx \). When \( x = 0 \), \( u = 9 \), and when \( x = 4 \), \( u = 25 \).

\[
\int_0^4 x \sqrt{x^2 + 9} \, dx = \frac{1}{2} \int_9^{25} u^{0.5} \, du = \left. \frac{1}{3} u^{3/2} \right|_{u=9}^{25} = \frac{1}{3} (125 - 27) = 32.667.
\]

From the approximations above, we find that the midpoint rule underestimates by 0.667\%, while the trapezoid rule over estimates by 1.33\%.

12. a. The solution of the Malthusian growth model is \( p(t) = 400 e^{rt} \). From the 4-week data, we have \( p(4) = 1328 = 400 e^{4r} \), so \( r = 0.25 \ln \left( \frac{1328}{400} \right) \approx 0.299999 \). The doubling time satisfies \( 400 e^{rt} = 800 \), so \( t = \frac{1}{r} \ln(2) \approx 2.3106 \) weeks.
b. By separation of variables, the differential equation can be solved

\[ \int \frac{dp}{p} = \int (a - bt) \, dt \quad \text{or} \quad \ln |p| = at - \frac{bt^2}{2} + C. \]

With the initial condition, the solution becomes

\[ p(t) = 400 e^{at - bt^2/2}. \]

We use the log form of the equation above to find the parameters \( a \) and \( b \). At \( t = 2 \) and \( 4 \), we can write

\[ 2a - 2b = \ln(805.5) - \ln(400) = 0.7000 \]
\[ 4a - 8b = \ln(1328) - \ln(400) = 1.200 \]

Subtracting 0.5 times the second equation from the first equation gives \( 2b = 0.1 \) or \( b = 0.05 \). This substituted into the first equation gives \( 2a = 0.8 \) or \( a = 0.4 \). Thus, the solution of the differential equation is

\[ p(t) = 400 e^{0.4t - 0.025t^2}. \]

The maximum occurs when \( 0.4 - 0.05t = 0 \) or \( t = 8 \) weeks. This gives a maximum population of \( p(8) = 400 e^{1.6} = 1981.2 \).

13. The equilibria of this population model satisfy \( P \left( 9 - 0.01(P - 70)^2 \right) = 0 \). Thus, \( P_e = 0, 40, \) and \( 100 \). From the phase portrait below, it is easy to see that the equilibria \( P_e = 0 \) and \( 100 \) are stable, while \( P_e = 40 \) is unstable. The carrying capacity for this population is \( P_e = 100 \), and the critical threshold number of animals required to avoid extinction is \( P_e = 40 \).
14. a. The derivative is

$$B'(t) = -\frac{8}{(t + 1)^3} + 1.$$ 

This derivative is zero when $(t + 1)^3 = 8$ or $t = 1$, so $B(1) = 2$. Checking the endpoints, we find $B(0) = 4$ and $B(3) = 3.25$. Thus, the minimum occurs at $(1, 2)$ and the maximum is at $(0, 4)$. A sketch of the graph is below.

b. The average hourly brain wave power during the test is

$$\frac{1}{3} \int_0^3 (4(t + 1)^{-2} + t) \, dt = \frac{1}{3} \left( \int_1^4 4u^{-2} \, du + \int_0^3 t \, dt \right),$$

$$= \frac{1}{3} \left( -4u^{-1} \bigg|_{u=1}^{u=4} + \frac{t^2}{2} \bigg|_{t=0}^{t=3} \right)$$

$$= \frac{1}{3} \left( -1 + 4 + \frac{9}{2} \right) = \frac{5}{2}$$

where we let $u = t + 1$ in the first integral above.
15. a. This is a standard logistic growth model, so the equilibria are $F_e = 0$ and 200 (thousand). Below is a sketch of the function with the phase portrait. The equilibrium $F_e = 0$ is unstable, while the carrying capacity, $F_e = 200$ (thousand), is a stable equilibrium.
b. With harvesting, the right hand side of the differential equation is written

\[ 0.4F \left( 1 - \frac{F}{200} \right) - 15 = -0.002F^2 + 0.4F - 15 = -0.002(F - 50)(F - 150). \]

It follows that the equilibria are \( F_e = 50 \) and 150 (thousand). Above is a sketch of the function with the phase portrait. The equilibrium \( F_e = 50 \) (thousand) is the critical number of fish needed to avoid extinction and this equilibrium is unstable. The carrying capacity, \( F_e = 150 \) (thousand), is a stable equilibrium.

16. a. This is a linear differential equation. Let \( z(t) = w(t) - 65 \), so \( \frac{dz}{dt} = -0.07z \) with \( z(0) = -62 \). Thus, \( z(t) = -62e^{-0.07t} \) or \( w(t) = 65 - 62e^{-0.07t} \). The graph is shown below. The weight of the 4 year old is \( w(4) = 65 - 62e^{-28} = 18.14 \) kg, while the girl weighing 30 kg satisfies \( 65 - 62e^{-0.07t} = 30 \) or \( e^{0.07t} = \frac{62}{35} \) or is \( t = \frac{100}{7} \ln \left( \frac{62}{35} \right) \simeq 8.17 \) years old.

\[
\begin{align*}
\text{b. This is a time varying differential equation, so} & \quad P(t) = \int \left( 200e^{-0.4t}(65 - 62e^{-0.07t}) \right) \, dt \\
& \quad = \int \left( 13000e^{-0.4t} - 12400e^{-0.47t} \right) \, dt \\
& \quad = -32500e^{-0.4t} + 26383e^{-0.47t} + C
\end{align*}
\]

With the initial condition, \( P(0) = 0 \), we have \( C = 6117 \), so

\[ P(t) = 26383e^{-0.47t} - 32500e^{-0.4t} + 6117. \]

The total amount of lead in the girl age 4 is \( P(4) = 3581 \mu g \) and in the girl age 10 is \( P(10) = 5762 \mu g \). (Asymptotically, \( P(t) \to 6117 \mu g \).)
c. Assuming the lead is uniformly distributed in the body, the concentration satisfies \( c(t) = 0.1P(t)/w(t) \). It follows that the concentration in the girl at ages 4 and 10 are \( c(4) = 0.1(3581)/18.14 = 19.74 \ \mu g/dl \) and \( c(10) = 0.1(5762)/34.21 = 16.84 \ \mu g/dl \).

17. a. The maximum population for \( P(t) = 54 + 24 t - 4 t^2 \) is found by differentiating with \( P'(t) = 24 - 8 t \), which is zero at \( t = 3 \). This gives a maximum population of \( P(3) = 90 \).

b. The maximum population for \( Q(t) = 54 + 34 \sin \left( \frac{\pi}{6} t \right) \) occurs when \( t = 3 \), which is when the argument of the sine function is at \( \pi/2 \). This gives a maximum of \( Q(3) = 88 \). A graph of the function is below.

c. For comparison, the average of the data is \( \frac{54+73+85+89+86+75+53}{7} = 73.57 \). The averages from the integrals are

\[
P_{ave} = \frac{1}{6} \int_{0}^{6} P(t) dt = \frac{1}{6} \int_{0}^{6} 54 + 24 t - 4 t^2 dt
\]

\[
= \frac{1}{6} \left( 54 t + 12 t^2 - \frac{4}{3} t^3 \right) \bigg|_{x=0}^{6}
\]

\[
= \frac{1}{6} \left( 54(6) + 72(6) - 48(6) \right) = 78
\]

and

\[
Q_{ave} = \frac{1}{6} \int_{0}^{6} Q(t) dt = \frac{1}{6} \int_{0}^{6} 54 + 34 \sin(1/6 \pi t) dt
\]

\[
= \frac{1}{6} \left( 54 t - \frac{204}{\pi} \cos(1/6 \pi t) \right) \bigg|_{x=0}^{6}
\]

\[
= \frac{1}{6} \left( 54(6) - \frac{204}{\pi} \cos(\pi) - \cos(0) \right) = 54 + \frac{68}{\pi} = 75.65.
\]
18. a. For this problem $\Delta t = 2$ and the data provide the endpoints for the Trapezoid rule. It follows that the integral is approximated by

$$
\frac{1}{2}c(t_0) + \sum_{i=1}^{10} c(t_i) + \frac{1}{2}c(t_{11}) \Delta t = (0 + 18 + 29 + ... + 0) \cdot 2 = 246.
$$

b. The maximum concentration for $c(t) = 0.038t^3 - 1.3t^2 + 11t$ is found by differentiating with $c'(t) = 0.114t^2 - 2.6t + 11$, which by the quadratic formula is zero at $t = 5.611$ hr and $t = 17.196$ hr. This gives a maximum concentration of $c(5.611) = 27.505$ g/l. Computing the integral for the polynomial fit, we find

$$
\int_0^{20} (0.038 t^3 - 1.3 t^2 + 11 t) dt = \left( \frac{0.038 t^4}{4} - \frac{1.3 t^3}{3} + \frac{11 t^2}{2} \right) \bigg|_0^{20} = 253.3.
$$

c. The maximum concentration for $c(t) = 10 t e^{-0.02 t^2}$ is found by differentiating with $c'(t) = 10 t e^{-0.02 t^2}(-0.04 t) + 10 e^{-0.02 t^2} (25 - t^2)$, which is zero at $t = \pm 5$ hr. This gives a maximum concentration of $c(5) = 30.33$ g/l. Computing the integral for the exponential fit by letting $u = -0.02 t^2$ with $du = -0.04 t dt$ and taking the endpoints $u = 0$ when $t = 0$ and $u = -8$ when $t = 20$, we find

$$
\int_0^{20} 10 t e^{-0.02 t^2} dt = -250 \int_0^{-8} e^u du = -250 e^u \bigg|_{u=0}^{u=-8} = 250(1 - e^{-8}) = 249.9
$$

d. The second model is better because it continues close to zero, while the polynomial tends to infinity. If a particular model is shown to be valid, then fewer blood samples would need to be drawn to get a reasonable fit to the data and approximate the drug dosage.