1. Approximate the solutions to the following equations using 3 Newton iterates starting at $x_0 = 1$. Show all 3 iterates.

   a. $x^3 = 5$,
   b. $x^4 = 13$.

2. Consider the function

   \[ f(x) = 4 + 8x^2 - x^4. \]

   a. Find the derivative of $f(x)$ and the second derivative, $f''(x)$.

   b. Find the $y$-intercept. Determine any maxima or minima and all points of inflection for $f(x)$. Give both the $x$ and $y$ values.

   c. Sketch the graph of $f(x)$. Is this function odd or even or neither?

   d. One of the $x$-intercepts is near $x = 3$. Use Newton’s Method starting with $x_0 = 3$ and performing two iterations to get a good approximation to this $x$-intercept.

3. Consider the function

   \[ f(x) = x^3 - 3x - 3. \]

   a. Find all extrema and points of inflection, giving both the $x$ and $y$ values.

   b. Sketch the graph of $f(x)$. Is this function odd or even or neither?

   c. Use Newton’s Method to approximate the value of the $x$-intercept. Start with $x_0 = 2$ and perform two iterations.
4. Let \( f(x) = 4\ln(x) - x \).

   a. Differentiate \( f \) and find any critical points. Determine the domain of \( f \) and sketch the graph.

   b. Use Newton’s Method to approximate the value of the \( x \)-intercept. Start with \( x_0 = 1 \) and perform two iterations to give the approximation of one zero. Then let \( x_0 = 8 \) and find \( x_1 \) to approximate the other zero.

5. A mass at the end of a spring oscillates about its equilibrium position (\( y = 0 \)), executing a damped harmonic motion. Suppose that the position of the mass is given by the equation

\[ y(t) = 2e^{-0.2t}\cos(2t). \]

   a. Find the velocity of the spring \( v(t) = y'(t) \).

   b. It begins at its maximum displacement, \( y(0) = 2 \) cm. The minimum displacement occurs at the first time when \( v(t) = 0 \). Use two Newton’s iterates with \( t_0 = 1 \) to approximate the time when this minimum displacement occurs, \( t_2 \). Find the approximate minimum displacement by evaluating \( y(t_2) \).