Spring 2003 HW Solutions Differentiation of Trigonometric Functions

1. $f'(x) = -12\cos(3x)$

2. $f'(x) = -14\sin(7x) - 2x$

3. $f'(x) = -18\cos^2(2x)\sin(2x)$.

4. $f'(x) = \dfrac{3\cos(3x)}{2+\sin(3x)}$.

5. $f'(x) = 2e^{2x}\left(\cos(4x) - 2\sin(4x)\right)$.

6. $f'(x) = 2x^2\cos(x^2-\pi) + \sin(x^2-\pi)$.

7. $f'(x) = 4\left(x^2 - \cos(2x^3)\right)^3\left(2x + 6x^2\sin(2x^3)\right)$.

8. $f'(x) = 12\cos^2(4x)\sin^2(4x) - 4\sin^4(4x)$.

9. $f'(x) = \dfrac{-8x\cos(x^2)}{\sin^5(x^2)}$.

10. $f'(x) = \dfrac{3 - 6\sin(3x)}{(2-\sin(3x))^2}$, using $\sin^2(3x) + \cos^2(3x) = 1$.

11. $y' = e^x(\cos(x) - \sin(x))$, so there is relative maximum at $\left(\frac{\pi}{4}, e^{\pi/4}/\sqrt{2}\right) \simeq (0.7854, 1.551)$ and a minimum at $\left(\frac{5\pi}{4}, -e^{5\pi/4}/\sqrt{2}\right) \simeq (3.927, -35.89)$. There is an absolute maximum at the endpoint $x = 2\pi$ with $y = e^{2\pi} \simeq 535.5$. Below is the graph.

12. $y' = \frac{-2\cos(2x)}{\sin^2(2x)}$. The period is π. The relative minima for $x \in [0, 2\pi]$ are $(\pi/4, 1)$ and $(5\pi/4, 1)$, and the relative maxima are $(3\pi/4, -1)$ and $(7\pi/4, -1)$. There are vertical asymptotes at $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, \pi, , 2\pi$. Below is the graph.

13. a. The maximum displacements occur with $y(t) = 2$ cm at times $t = \frac{n\pi}{5}$, where $n = 0, 1, 2, ...$, and the minimum displacements occur with $y(t) = -2$ cm at times $t = \frac{\pi}{10} + \frac{n\pi}{5}$, where $n = 0, 1, 2, ...$ The period is $T = \frac{\pi}{5} \simeq 0.6283$ sec.

b. The velocity is $v(t) = -20\sin(10t)$, and the acceleration is $a(t) = -200\cos(10t)$. The maximum velocity is 20 cm/sec occurring at $t = \frac{3\pi}{20} + \frac{n\pi}{5}$ sec.

14. a. The volume of air satisfies $V(t) = 2500 + 300\cos(2\pi t/3)$.

b. The derivative is $V'(t) = -200\pi\sin(2\pi t/3)$.The maximum exhalation in $200\pi \simeq 628.3$ ml/sec occurring at $t = \frac{3}{4}$ sec.

c. The graphs are below.

15. a. $F(t) = 0$ when $t = \frac{n\pi}{6}$ sec. Foot on the ground for $\frac{\pi}{6}$ sec.

b. $F'(t) = A\left(b\cos(bt) - ab\left(3\sin(bt)\cos(3bt) + \cos(bt)\sin(3bt)\right)\right)$. $F'\left(\frac{\pi}{12}\right) = 0$, since $\cos\left(6\frac{\pi}{12}\right) = 0$ and $\cos\left(18\frac{\pi}{12}\right) = 0$. The maximum value is $F\left(\frac{\pi}{12}\right) = A(1 + a)$

16. a. Since $\sin(\theta) = 4/d$, so $d = 4/\sin(\theta)$.

$$I = k\frac{\cos(\theta)}{d^2} = k\frac{\cos(\theta)}{(4/\sin(\theta))^2} = \frac{k}{16}\cos(\theta)\sin^2(\theta).$$

Differentiating

$$I'(\theta) = \frac{k\sin(\theta)}{16}\left(2\cos^2(\theta) - \sin^2(\theta)\right).$$

b. When $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \sqrt{2}$, then $\sin^2(\theta) = 2\cos^2(\theta)$, so $I'(\theta) = 0$. The optimal height is $h = \frac{4}{\sqrt{2}} \simeq 2.83$ ft.

17. a.

$$\frac{dS}{d\theta} = \frac{3R^2}{2} \frac{1 - \sqrt{3}\cos(\theta)}{\sin^2(\theta)}.$$

b. $\cos(\theta) = \frac{1}{\sqrt{3}} \simeq 0.5774$ minimizes the surface area. It follows that $\arccos\left(\frac{1}{\sqrt{3}}\right) \simeq 0.9553 \simeq 54.7°$.