1. Evaluate the following definite integrals:

a. $\displaystyle\int_{-1}^{3}(2-x+x^2)dx$,

b. $\displaystyle\int_{0}^{4}\left(x^2+3-e^{-x}\right)dx$,

c. $\displaystyle\int_{0}^{2}\left(x^2+1\right)^2 dx$,

d. $\displaystyle\int_{0}^{3\pi}\cos(\tfrac{t}{2})dt$,

e. $\displaystyle\int_{-1}^{5}\frac{dx}{\sqrt{6+2x}}$,

f. $\displaystyle\int_{1}^{5}\frac{x^2+1}{x}dx$,

g. $\displaystyle\int_{0}^{\pi}(\cos(2t)+4t)\,dt$,

h. $\displaystyle\int_{0}^{\pi/2}\frac{\cos(x)}{1+\sin(x)}dx$,

i. $\displaystyle\int_{3}^{4}\frac{2x}{\sqrt{25-x^2}}dx$,

j. $\displaystyle\int_{0}^{\pi}\left(3t^2-\cos(3t)\right)dt$,

k. $\displaystyle\int_{-2}^{2}\left(\frac{1}{x+3}+e^{2x}\right)dx$,

l. $\displaystyle\int_{0}^{\pi/2}3\left(\sin(x)+1\right)^2\cos(x)dx$.

2. Find the area bounded by the function $y = 4 - x^2$ and the x-axis. Sketch the graph of the region.

3. Find the area between the function $y = 3\sin(2x)$ and the x-axis for $0 \leq x \leq \pi/2$. Sketch the graph of the region.

4. Consider the curves $y = x + 3$ and $y = x^2 + x - 6$.

a. Sketch the graph of these curves. Show the x and y-intercepts and any vertices.

b. Find the points of intersection of these curves. (Hint: Set the equations equal to each other and solve for x.)

c. Find the area between the two curves.

5. Consider the curves $y = -2x - 1$ and $y = 15 - 2x - x^2$.

a. Sketch the graph of these curves. Show the x and y-intercepts and any vertices for both curves.

b. Find the points of intersection of these curves. (Hint: Set the equations equal to each other and solve for x.)

c. Find the area between the two curves.

6. Consider the curves $y = \sqrt{x}$ and $y = x^3$.

a. Sketch the graph of these curves. Find the points of intersection of these curves.

b. Find the area between the two curves.

7. Consider the region bounded by the curves:

$$y = x^2 \qquad \text{and} \qquad y = 8 - x^2.$$

Sketch the region and find the area between the curves.

8. Find the area in the first quadrant that is bounded between the curves:

$$y = \sqrt{x}, \qquad y = \frac{1}{x}, \quad \text{and} \quad y = \frac{x}{4}.$$

Sketch the graph of the curves showing all points of intersection.

9. Two researchers analyze seven years of population data for a particular animal that is given in the table below.

Year	0	1	2	3	4	5	6	7
Pop	12	18	27	32	28	17	12	21

a. The first researcher fits the data with the quartic equation

$$P(t) = \frac{1}{4}t^4 - 3t^3 + 9t^2 + 12.$$

Find the minimum and maximum populations using this approximation to the data. (Show how you compute the extrema.)

b. Find the average population by computing the definite integral

$$P_{ave} = \frac{1}{7}\int_0^7 P(t)dt.$$

c. The second researcher fits the data with the curve

$$Q(t) = 22 - 10\cos\left(\frac{\pi}{3}t\right).$$

Sketch a graph of this curve and give where the minimum and maximum populations occur.

d. Find the average population by computing the definite integral

$$Q_{ave} = \frac{1}{7}\int_0^7 Q(t)dt.$$

10. Below are six years of data from some particular animal population (in thousands):

Year	0	1	2	3	4	5	6
Pop	53	37	39	54	70	68	52

a. These data are fitted pretty well by the function

$$P(t) = 53 - 18\sin\left(\frac{\pi}{3}t\right).$$

Sketch a graph of this curve and show the data points from the table. Use this function to find when the maximum and minimum populations occur and what their values might be.

b. Find the average population by computing

$$P_{ave} = \frac{1}{6}\int_0^6 P(t)dt.$$

11. One of the hazards of modern medicine is the possible exposure to radioactive sources used in cancer treatment. Radioactive iodine, ^{131}I, is used in the treatment of certain thyroid problems.

a. A differential equation describing the radioactive decay of ^{131}I is given by

$$\frac{dR}{dt} = -0.1R, \qquad R(0) = 50,$$

where t is in days. Solve this differential equation and find the half-life for ^{131}I.

b. Suppose a technician is receiving an exposure from mislaid sample of

$$D(t) = 5e^{-0.1t},$$

in mCi/day. The total exposure over 10 days is given by the integral

$$\int_0^{10} D(t)\,dt.$$

Find this total exposure.

c. How long can the technician stay near this source if the exposure is to be kept to less than 10 mCi?

12. a. An agricultural pest is experiencing an outbreak that satisfies the Malthusian growth law

$$\frac{dP}{dt} = kP, \qquad P(0) = 100,$$

where t is in weeks. If it is observed that the population of this pest is 250 after one week, then determine the doubling time of this pest and find a general expression for the population as a function of time.

b. The average population over a period of 4 weeks is given by the integral

$$\frac{1}{4}\int_0^4 P(t)\,dt.$$

Find the average population of this pest over this period of time.

13. The **normal distribution**, which is commonly used in statistics, for a random variable x is given by the formula

$$N(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/(2\sigma^2)},$$

where σ is the standard deviation. The expected value of x over an interval $[a, b]$ is given by the formula

$$\int_a^b x\,N(x)\,dx.$$

This can be interpreted as the average value of x that one expects over the interval. Find the expected value of x if $\sigma = 1$ and the interval is $[0, 2]$.